# THE PHYSICS SOCIETY

## ANNUAL PHYSICS JOURNAL

## St.Stephen's College, Delhi

ECHO JOURNAL

## THE PHYSICS SOCIETY ST.STEPHEN'S COLLEGE

DELHI



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# CREDIESHO ECHO

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### Message from the staff adviser,

St. Stephen's college has several societies; together the only purpose of all of them is to provide space to the junior members to explore things beyond the rigid curriculum framework and classroom disciplines. The presence of this space is essential in the development of a rational, mindful and healthy human being. Junior and senior members interact with learned people from academia and research institutes through talks, lecture series and discussion forum etc.

Physics Society of the St. Stephen's College plays its role very effectively and efficiently in this bigger picture Started primarily with the Feynman club, Problem solving club, Astronomy club etc. the Physics Society has expanded over the years in order to include new fields because of the growing demand and nature of problems that requires interdisciplinary approach. Physics Society organizes talks, lecture series and other event where scientists, academicians, or even their fellow classmates share their knowledge on the topics from diverse fields, pure and interdisciplinary. This is possible only because of the love, passion and enthusiasm of the students for the society and it in turn comes from the impacts of these events in their professional and academic spheres. The Physics Society achieved all these after a very well thought and much deliberated process such that the time and resources of the society will be utilized efficiently.

The current pandemic has affected the way these events were organized in the past. It left no choice for us to organize the events in our traditional way. The Physics Society, however, found out the solution and used the online platform to continue with the activities even in the lockdown period and in the duration when the academic work is halted because of the pandemic. It is commendable and I congratulate every member of the Physics Society for their efforts.

In the end, I wish to thank and congratulate every member of the physics department, students as well as my colleagues, who have helped and contributed to the successful completion of the activities of physics society for the current academic year. I congratulate every members of the Physics Society and appreciate their effort in organizing various events and to the publication of this year journal. I wish them my very best for their future endeavors.

Dr. Harish Kumar Yadav Staff Adviser

#### FROM THE EDITOR'S DESK

#### " Forsan et haec olim meminisee iuvabit. "

Aeneas tells his exhausted, shipwrecked followers in "The Aeneid," Book 1. "Maybe someday, you will rejoice to recall even this." We may have quoted this line in a desperate attempt to look scholarly but the spirit of the phrase remains intact. The year that has gone by was a tough one ('tough' being one of the more respectful adjectives!). With the world cooped up in their homes to fight the horrors of a virus that even the most far-sighted of men hadn't expected!

Contrary to what science tells us, this uncertainty wasn't beautiful. Despite lives lost around the world and so many adverse effects faced, we will selfishly limit myself to our experiences as a department and society.

The previous versions of 'Echo' focused on firing up the passion for science in our students. It aimed at encouraging them to pursue and present their research interests. What it celebrates is the power of intuition, knowledge and sometimes 'sweet cluelessness'.

The aim of this year's journal remains the same except a slight little addition. This year we also celebrate how we came together as a department and tackled the adversities of these times to the best of our capabilities.

From our professors giving their absolute best to help us warm up to the concept of online education to our students coming up with new and innovative ways of continuing society activities. From whining to each other about the absolute cumbersomeness of online classes to long, passionate discussions on google meet about physics, research, life and impending future. From welcoming the first years to sympathizing with them over having missed out on all the fun and important experiences. We indeed have come a long, long way. We might not be looking back with joy at the gloominess of this lockdown but the way it bought all of us closer together is indeed to be cherished.

The journey an individual takes with physics is one you can learn much from, beyond what you learn in the syllabus. The way in which we unearth it and discover our awe for the subject is by itself a beautiful story. You learn a bit more about yourself, the differences in the way you think, because after all none of us are the same and you ultimately learn that, that isn't wrong. Physics is surprisingly about creativity and hence there is no fixed way to think and to realise that you are just different is liberating. We need diversity (at least we tell ourselves this). Working for the journal made us reiterate the fact that, "the joy of discovery can only be found at a crossroad", all this while we were searching for the romantic perception of physics in the subject alone, but the discovery of literature as a way to express our love for physics, brought back the spark that we were searching for and we sincerely hope that you have found what you were searching for or will find it soon.

This journal is an ode to the heart and the head. With extremely well-written research articles from guest authors and our very own students to deeply personal experiences to doodles and memes that made us smile and pictures that will hold a permanent place in our memories, this journal is an encomium to us and our uncrushable pursuit of knowledge. And lastly, there a lot of people who deserve to be thanked because without them this endeavour would have ended in shambles. We are indebted to our staff adviser Dr. Harish Kumar Yadav for his guidance and support. We are grateful to our president Neelam Firdous Khan and other members of the council for their constant help and encouragement. We are indebted to everyone who submitted their work for the journal and worked hard to make it a better one.

We would like to end with another Latin quote which we hope inspires you to never give up on this adventure of learning.

#### 'SAPERE AUDE'

(dare to know!)

-Shalika Yekkar and Swapnila Chakrabarty

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#### **Physics Society Events :2020-21**

Science has been valued by man for years. Going back in history, the Greeks never differentiated the lines that now divide the humanities and sciences but now we are going back and forth. We define a new class of multidisciplinary subjects that gives us the understanding that we cannot put subjects into concrete boxes but that different approaches help us gain a better understanding of the subject itself. With 2020 being an unprecedented one, we have tried our best to adapt to the new 'normals' and make the best of it, the summary report of which is presented below.

#### **Astronomy Club**

This year has been a momentous year for the Astronomy club. We have accomplished a lot starting from establishing new verticals to rediscovering interesting astronomical results from computational simulations. From the age-old sky viewing sessions to the computational analysis of the N-body problem, we haven't let even a single aspect of Astronomy unexplored. Radio Astronomy, a vertical under the Astronomy Club facilitates under-graduate students in understanding and working with EM Simulations to collect data and using RTL-SDR paired up with a dipole antenna or horn antenna we fetched radio data for analysis. Speaking of collecting and analysing data, we also had the Data Analysis team which undertook the following studies on "Studying variation of Total solar irradiation and the sunspot number over the solar cycle" and "Analysis of oscillatory behaviour in H-Alpha intensities of solar flares and sunspots of chromosphere".



Having looked at the sun, the STAR of our solar system, we undertook a stellar dynamics project to look at restricted 3-body problem (a binary star system), the resonance condition of orbits, kozai-lidov instability and even interesting features like "Choreography". Finally, in order to generalise the problems some third year students picked up the N-body problem and undertook a rigorous mathematical analysis of the adaptive refinement and domain meshing. Moving on to cosmological implications of different initial mass distributions and finally analysing a more robust version by incorporating different physics modules from GENGA code and running the program simultaneous on CUDA.

#### **Problem Solving Club**

The Problem Solving Club of the Physics Society at St. Stephen's college aims to provide students with a platform to discuss Physics problems beyond the realms of the standard curriculum. The club aims to inculcate a sense of curiosity in physics and mathematical sciences by encouraging students to improve their analytical and computational skills. This year the problems discussed ranged from topics such as minimal surfaces, electrostatics in different spaces, nearest neighbour distributions and a few more. We held our brainstorming sessions online and had a decent turn up for most of them. These sessions were used as a platform for students to come up with rather creative suggestions and solutions

to the problems posed. We hope the club grows and continues the tradition of coming up with creative solutions to challenging problems.

#### **Feynman Club**

The Feynman Club of The Physics Society provides students a platform to have an understanding of research in science in the best way possible through interactions with eminent and experienced physicists and introduction to a wide spectrum of domains in physics through research papers through the Journal Reading Group solely dedicated to student discussions on interesting research papers were discussed on a weekly basis. The Following Feynman talks were organised on online platforms:

| Sl.No | Date        | Speaker  | Title  |
|-------|-------------|--|--|
| 1     | 21 Aug 2020 | Dr. Nissim Kanekar<br>NCRA , TIFR, Pune , India.                                   | Do the Fundamental constants change with time??                    |
| 2     | 28 Aug 2020 | Dr. Suvrat Raju<br>ICTS, TIFR, Bengaluru   | How close are pure states to thermal states??                      |
| 3     | 4 Sep 2020  | Dr. Arijit Pal<br>Dept of Physics and Astronomy, University College, London.       | Many-body localization: Breakdown of ergodicity in quantum matter  |
| 4     | 18 Sep 2020 | Sarthak Vijay<br>III year B.Sc(H) Physics  | A proof of the Crystallographic Restriction Theorem                |
| 5     | 9 Oct 2020  | Dr.R Vijayaraghavan<br>Professor, TIFR.  | Quantum computers for physics, chemistry, biology, and more        |
| 6     | 6 Nov 2020  | Prof. Sanjay P. Sane<br>NCBS, TIFR, Bengaluru, India.                              | Fast, small - yet still in control: the mechanics of insect flight |
| 7     | 8 Jan 2021  | Dr. Bikram Phookun<br>St.Stephen's College, Delhi.                                 | Studying Physics in the 21st Century                               |
| 8     | 29 Jan 2021 | Kesav Saranyan Krishnan<br>PhD student, University of Illinois at Urbana-Champaign | An Introduction to Universality                                    |

#### How to Explore Venus' Interior without Landing on It

Siddharth Krishnamoorthy, NASA Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA

It may surprise some to learn that Venus was the first planet to be visited by a spacecraft (Mariner 2, USA) and the first to experience a soft landing by the human race (Venera 7). For nearly three decades, Venus enjoyed significant scientific attention, from the United States as well as the former Soviet Union, which successfully deployed ten landers on its surface and even floated two balloons in its atmosphere. Prior to these early missions, Venus was a planet shrouded in mystery, among the brightest objects in the Earth's night sky, but also entirely shrouded in thick clouds that obscured any view of its surface. Early flyby and landed missions discovered a planet with an atmosphere predominantly made up of CO<sub>2</sub> and extreme temperature (>460°C) and pressure (90 atmospheres) at the surface, which caused all of the landers to thermally fail within 2-3 hours after landing. This also led to the discovery of the greenhouse gas effect and eventually the prediction and observation of the same effect on Earth from anthropogenic emissions. The two balloons (Vega-1 and 2) floated by the Soviet Union in 1985 were (and will be, until the Mars Helicopter takes flight on Mars in 2021) the only aerial vehicles ever deployed on another planet. They discovered that the dense clouds of Venus were made primarily of sulphuric acid, but the temperature and pressure environment at approximately 55-60 km altitude was remarkably Earth-like (0°C and 1 atmosphere pressure). Owing to these conditions, both balloons survived for two Earth-days (after which the batteries were exhausted, the balloons may still have been operational), a factor of 20 longer than any of the landers. The Vega balloons also discovered that Venus' atmosphere rotates around the planet approximately once every four Earth days, which is nearly 30 times faster than the planet's rotation rate about its own axis. In fact, the thickness of Venus' atmosphere combined with its high velocity relative to the ground alters the length of day on Venus and is a major driver of the planet's climate.

In the 1990s, NASA's Magellan spacecraft used radar to image Venus' surface (since the clouds are optically opaque) and revealed a planet covered in volcanic and seismic landforms. Since the 1990s, there has been no NASA mission to Venus. However, other space agencies have sent spacecraft such as Venus Express (European Space Agency) and Akatsuki (Japanese Aerospace Exploration Agency), which have investigated the dynamics of Venus' atmosphere. In the last decade, however, Venus has increasingly been recognized as an enigmatic planet ignored, and interest in its exploration is increasing. Owing to challenging conditions on its surface, balloons and orbiters have emerged as important vehicles for Venus exploration. There have also been efforts to develop high-temperature technology that will allow for longer surface operation. NASA is currently considering two missions to Venus as part of its Discovery mission competition. India recently announced the Shukrayaan orbiter mission to Venus, currently scheduled for a 2024 launch. The European Space Agency also has plans to continue the exploration of Venus. Russia, which discontinued Venus exploration after severe budget shortages following the fall of the Soviet Union, has plans for a mission to Venus

with a long-lived lander and a balloon platform. Many in the planetary science community believe that the exploration of Venus will become a major priority in the coming decade.

#### Why is Seismology on Venus Important and Challenging?

Seismology on any rocky planet is key to understanding the dynamics of its interior. On Earth, large seismic networks deployed around the world record the motion of its surface to determine the seismic background and register arrivals from earthquakes, bolide impacts on the surface, chemical and nuclear explosions, and even major hurricanes. Over decades of research and analysis, these seismometer networks have led to the discovery of not just Earth's layered internal structure (crust, mantle, and core), but several smaller, fine-grained features in the sub-surface. In turn, knowledge of the Earth's interior has furthered our investigation into the early solar system and conditions that led to planetary formation and evolution, striking at the heart of arguably the most important question in planetary exploration - "Where did we come from?". As important as it is to understand the Earth's internal structure, the story of the solar system is incomplete without studying all its planets as a system. During the Apollo era, astronauts placed sensitive seismometers on the Moon, which recorded moonquakes and led to the conclusion that Moon is also layered and differentiated like Earth, except that core is very small (~350 km radius), indicating that the Moon has largely cooled down since its formation. In 2018, NASA sent the Interior Exploration using Seismic Investigations, Geodesy and Heat Transport (InSight) spacecraft to Mars, equipped with an extremely sensitive seismometer, to listen for seismic waves and diagnose the interior structure of Mars. In just over two years since then, InSight has recorded close to 500 marsguakes. mostly weak (magnitude <=4.5) and shallow, so that seismic waves have rarely propagated through the Martian mantle. Judging by the number and strength of quakes detected so far, Mars appears to violate the scaling laws followed by the number of guakes on Earth and the Moon. The mission was recently extended for two more (Earth) years, potentially expanding the catalogue of events available to seismologists for further study of Mars' interior.

In this system of terrestrial (rocky) planets and planet-like bodies, Venus is both scientifically mystifying and technologically challenging. With approximately 81% of the Earth's mass and a radius that is 95% of Earth's, it is structurally remarkably similar to the Earth. However, that is where all similarities stop. With 460 C and 90 atmosphere pressure, and an atmosphere that is 98% CO<sub>2</sub>, the Venusian surface is extremely inhospitable to life (although there has been substantial debate about life in its cloud layer after a recent discovery of phosphine gas). Venus also rotates in the opposite sense as Earth and most other planets, and extremely slowly – a solar day on Venus is 117 Earth days. It turns out, that we currently have very little idea of why any of the above has come to pass. It is largely thought that a visitor to the solar system approximately 3 billion years ago might have found three blue planets – Venus, Earth, and Mars, following which their evolutionary paths diverged dramatically.

Studying Venus' internal structure offers important clues to why its evolution differed so much from the Earth. However, the hostile conditions on its surface have posed a severe

technological challenge to performing an experiment such as InSight. As mentioned before, none of the landers sent by the Soviet Union from the 1960s to 1980s lasted longer than three hours, which is too short to perform seismology. Even though there are efforts underway to develop sensing technology that can withstand the harsh environment for long periods of time (~60 days), noise from surface winds, storing and communicating data at such high temperatures are additional challenges with performing seismology on Venus. Therefore, the timeline for performing an InSight-like mission on Venus is likely on the order of a few decades. However, where Venus' atmosphere stunts our ability to perform "traditional" seismology, it also enables other modes of seismology, never deployed anywhere in the solar system.

#### Seismology through Remote Sensing

On planetary bodies with atmospheres, seismic energy generated by quakes is transmitted into the atmosphere through coupling across the surface-atmosphere interface. The low-frequency component (0.001 to 20 Hz) of these atmospheric perturbations travel unattenuated for large distances on Earth. For pressure waves in the > 0.01 Hz frequency range, molecular repulsion acts as the restoring force, and gravity is practically constant over the wavelength. These waves are called acoustic waves. More specifically, acoustic waves with frequency below 20 Hz are called infrasound (the opposite of ultrasound). For yet lower frequency atmospheric perturbations (< 0.01 Hz), gravity and buoyancy act as a restoring force, and such waves are known as gravity waves (not to be confused with Einstein's gravitational waves). Infrasound from weak and strong quakes has been observed by barometers deployed on Earth's surface. In the case of large earthquakes, gravity waves have even been observed in the Earth's ionosphere, using GPS satellites.

The efficiency of surface-atmosphere coupling is governed by the difference in density of the atmosphere and the surface. Similar phenomena are observed in the propagation of electromagnetic waves across different media, where it is the difference in refractive index that governs how much energy is transmitted across the interface. With surface pressure at 90 atmospheres, the Venus atmosphere provides 60-times better acoustic impedance matching across the interface for seismic energy looking to propagate into the atmosphere, potentially resulting in much stronger atmospheric signatures than on Earth for the same quake magnitude. The key to performing seismology on Venus hinges on the observation of these acoustic waves to study seismic waves traversing its crust, not from the surface, but from the upper atmosphere and from orbit. Atmospheric conditions at approximately 60 km altitude have been demonstrated by the Vega balloons and subsequent experiments to be much more tolerable (~0°C) and much longer mission lifetimes can be guaranteed without the use of high-temperature electronics. In addition, atmospheric waves from stronger quakes may be observed from orbit, where mission lifetimes on the order of years are routine.

#### Measuring Seismic Activity from Balloons

The airborne measurement of seismic activity using its infrasound signature is a relatively new area of research, but rapid progress has been made through analog experiments on Earth using artificial seismic sources. There are three key capabilities that need to be established for this effort to be successful. First, it must be demonstrated that pressure waves from seismic activity, which have been observed on ground-based stations, are also observable on balloons. In addition, unlike a traditional seismometer, since the measurement of ground motion is indirect, the relationship between seismic and pressure signals must be studied carefully. Second, the ability to discriminate a seismically-induced pressure wave from all the other source of pressure fluctuations on a moving platform, either from other environmental sources such as wind gusts, or from the motion of the balloon, must be demonstrated. Lastly, balloons sent to other planets are severely constrained in the amount of payload they can carry and the amount of data they can transmit back to the Earth. Therefore, the mass and power requirements for the sensor used in this effort must be minimized, and the ability to autonomously identify and transmit "interesting" snippets of data must be demonstrated.

Recent experiments using a weak artificial seismic source known as a "seismic hammer" have shown that seismic activity is, in fact, detectable in the air from its infrasonic signature. Moreover, it was found that the frequency content of ground motion is imprinted into its atmospheric signature, meaning that the measurement of pressure in the air may be equivalent to measuring ground motion to within a multiplicative factor in the infrasound frequency regime. Quakes may also be located by deploying multiple barometers on a tether and measuring the time of flight of the signal between the barometers. With reliable motion tracking, pressure variations induced by the changing altitude of the balloon in a convective atmosphere can be removed. Additional experiments are planned using frequent balloon flights over seismically active areas on Earth to demonstrate this capability for natural earthquakes.

The detection limit of this technique is currently undetermined. Preliminary computations suggest that infrasound from quakes with surface magnitudes as low as 3.0 may be detected over 100 km away from the epicenter of a quake on Venus, if pressure



Figure 1: (Left) A schematic of the types of seismic waves that can be detected using barometers on a balloon [from Cutts et. al., 2021]. (Right) A solar-heated balloon carries a barometer aloft during an Earth-based test flight in 2019.

fluctuations as low as  $10^{-3}$  Pa can be accurately measured. For context, speech at conversational volume produces perturbations of ~0.01 Pa approximately 1 m away from the speaker. Once detected, the infrasound signature may be used to study several outstanding questions about Venus seismicity.

Differentiation of seismically generated acoustic waves from other sources of pressure fluctuations and automated event detection are challenges that can be effectively addressed by collecting and analyzing large balloon-based datasets on Earth (Figure 1). Infrasound signals in these datasets may be attributed to their originating events using other independent streams of "ground truth" available on Earth, then identifying unique features in the infrasound signal to enable autonomous event identification and discrimination.

#### Measuring Seismic Activity from Orbit

On Venus, partial ionization and excitation of CO<sub>2</sub> and O<sub>2</sub> molecules in the atmosphere leads to thermal emissions in the 4.28 µm and 1.27 µm wavelength "airglow" near-infrared optical bands at approximately 100 km altitude. In fact, Venus' airglow is the brightest in the solar system. When pressure waves from seismic activity cross this altitude, they perturb the density and temperature of neutral and ionized molecules. This results in a fluctuation of the airglow, not unlike ripples on the surface of a pond (Figure 2). A satellite in Venus orbit equipped with the ability to take images at these wavelengths and in an orbit that can image the entire planetary disk can track these airglow perturbations. By



Figure 2: A simulation of propagating seismic waves as visualized in the Venus airglow layer, seen from an orbiter in equatorial Venus orbit that can view the whole planetary disc (e.g., VAMOS) [from Didion et. al., 2018].

analyzing the travelling ripples in the airglow layer, the location of the quake can be found. Further, by looking at the frequency content of the airglow wave as it propagates across the planet, the underlying seismic structure can be investigated. This is the basis of the Venus Airglow Measurements and Orbiter for Seismicity (VAMOS), a concept that was envisaged as part of a NASA-funded study in 2017. This study analyzed the performance limits for such a satellite, including on-board data processing and automated identification of seismic events, and determined that quakes as low as magnitude 5.3 may be detected from orbit. Rayleigh waves generated by quakes as low as magnitude 5.5 on the dayside could be used to probe the mean thickness of Venus' crust. Although the balloon-based measurement technique is expected to be more sensitive to weaker quakes that occur in near the balloon (<100 km), the orbiter-based approach has the advantage of a global

view of the planetary disk. The orbiter platform can operate in coordination with the balloon platform, serving as a communication relay and a balloon tracking station, and providing global context to local balloon measurements. Thus, the orbiter and balloon techniques are complementary.

#### <u>A roadmap for Venus seismology</u>

Although measurements of seismic activity using surface landers is the most direct way to perform seismology, on Venus, technology that enables such an investigation is still decades away. As mentioned above, the balloon and orbiter-based techniques are able to perform seismology from a remote station, circumventing the need for high-temperature electronics. Each of the two techniques discussed here have seen rapid development in the last decade and can be performed with technology available today, in conjunction with other Venus balloon- and orbiter-based investigations (e.g, Venus atmospheric physics and chemistry). These techniques can determine the level of seismic and volcanic activity on Venus, identify locations of heightened activity for further investigations may also be viewed as the first missions to explore Venus' seismic and volcanic character, and pave the way for more advanced, long-lived seismometers deployed on the surface as part of highly targeted investigations to map the deep interior of Venus. The determination of Venus' internal structure is a key piece in the puzzle to understand the origin of the rocky planets, and in turn, the origin of our solar system.

#### About the author

Siddharth Krishnamoorthy is a Research Technologist in the lonospheric and Atmospheric Remote Sensing group at the NASA Jet Propulsion Laboratory (NASA-JPL) in Pasadena, California. Siddharth graduated his BSc physics degree from St. Stephen's College in 2007, following which he completed a MSc degree in physics at the Indian Institute of Technology Delhi (2009), where his thesis focused on simulations of the Random Field Ising Model (RFIM). He then completed MS (2012) and PhD (2017) degrees in aeronautics and astronautics from Stanford University. Upon graduation from Stanford University, he joined NASA-JPL as a postdoctoral associate, and transitioned to his role as a technologist in 2019. Siddharth's main research interest is in technology development for space exploration, which typically involves solving a combination of physics and engineering problems in severely resource-constrained environments. At NASA-JPL, his research is currently focused on developing infrasound remote sensing for Venus application. He is also the instrumentation lead for Venus balloon prototype development and contributes to teams working on ionospheric remote sensing for Deep Space Network (DSN) calibration and natural hazard detection and early-warning systems.

#### **Spontaneity and Creativity**

I dwell in Possibility — A fairer House than Prose — More numerous of Windows — Superior — for Doors — —Emily Dickinson

The subliminal self is in no way inferior to the conscious self; it is not purely automatic; it is capable of discernment; it has tact, delicacy; it knows how to choose, to divine.

—Henri Poincaré

I attended the Meera Memorial Paper Reading Competition for the first time a few months after joining College as a teacher in July 1996. Perhaps for this reason, I remember it more clearly than most of the others in the following twenty-five years. Sandeep Krishna, a third-year student, spoke on auto-catalytic reactions, which he had studied over the summer at IISc. Pranjal Trivedi, a secondyear student, spoke on the flash spectrum of the Sun, which he had photographed during the complete solar eclipse that took place in 1995. His classmate Advaith Siddharthan spoke on chaos (after having assured me that it wasn't going to be the concatenation of non sequiturs that popular talks on chaos can be). After the talks the judges went out, and there was a pause while the rest of us waited for their decision. I stepped up and without forethought said a few words about how to give a good talk. I said: always prepare, or you'll produce nothing but hot air; on the other hadn't, don't over-prepare, or your words will fall out like rocks: be prepared, then let go, and you'll be able to speak straight. After the talk, one of my old teachers, Dr Swaminathan, approached me, and said, in some amazement, "Bikram, I didn't know that you were an orator!" On hearing this I was embarrassed and amused, and I no doubt asked myself how I had reached this point. As a child and right into my college days I was extraordinarily shy. Though able to speak fluently with friends, family, and teachers, I became completely tongue-tied when I had to speak in public. I have told the story of my journey in public-speaking in *Learning to Speak*, and I won't go into it here. (It's available on my blog, feuilleautumn.wordpress.com.)

What I wish to focus on is the formula I offered: prepare, let go, speak. When you are completely unprepared, your mind is in a state of chaos and it's difficult for you to be coherent. On the other hand, when you are over-prepared your script is frozen in your mind, and you are unable to react to the needs of the moment. To be spontaneous you need to let go of what you have prepared — but unless you have prepared something you have nothing to let go of. When you prepare, you create a mental entity. When your let go of it, it doesn't disperse into nothingness — instead, it takes on a life of its own. And unless what you say is inspired by that breath of life it falls flat. The conscious act of preparing alone can never give it life. This — the infusion of life — happens in the interregnum between preparation and delivery. So another version of my formula would be: prepare, wait, speak. For the letting go often happens naturally while we wait and allow ourselves to be occupied with other matters — exercise, day-dreaming, sleep, play, even superficialities and distractions.

When we engage intensely with something purposeful, e.g. preparing a talk, our conscious mind takes charge. But if, having prepared, we let go, what we have put together enters our subconscious, and it is there that it acquires what I call life. The subconscious offers possibilities for reconfiguration and animation than the conscious mind does not. When you deliver your talk thereafter you draw not directly on what the conscious mind had originally prepared but on what remerges into it after a period of immersion in the subconscious. It's important to understand that both stages — preparation by the conscious mind animation by the subconscious — are equally important.

Those of us who teach are very familiar with the need for preparing our lectures if we're to be make sense. On rare occasions, we may risk turning up for a class un-prepared. But being unprepared for a whole course is unthinkable for most of us. (There was, many years ago, a famous teacher of ancient Indian history at St Stephen's by the name of Mr P S Dwivedi, who knew his subject so intimately that he apparently always just spoke spontaneously in the classroom, and everything he said was interesting and enlightening; asked once if he prepared for his classes, he said, "I am prepared for every lecture that I have ever given or will ever give." But most of us are not like that.) One year – this was in the annual mode – when teaching introductory Electricity & Magnetism after having taught it several times in the past, I decided to try *not* preparing immediately beforehand, using instead whatever I carried in my head from earlier years. I noticed that whereas I was sometimes less coherent than usual, at other times the lesson would come together almost magically. In other words the fluctuations were much larger than usual. On one occasion, while doing something in magnetism, I got stuck, and, perhaps to cover up for my embarrassment, I said to the students, "Have you guys figured out that I have been teaching this course unprepared?" A student by the name of Philip Cherian gave me a response that will stay with me till my dying day: "Yes, sir: you weren't doing very much." Not doing very much! I was stunned, and immediately went back to my usual pre-lecture preparation, and the course went off well. I learned a couple of important lessons from this experience: first, that I needed to do my usual pre-lecture preparation diligently, and second, that I needed to let go enough for fluctuations to grow. Too much letting go, or too great an interval since preparation, and the fluctuations would grow into chaos; too much preparation, or too short an interval since preparation, and the fluctuations would fail to quicken. Spontaneity meant finding the sweet spot in between.

My formula for spontaneity brings me to something closely related to it, creativity. Consider an area where doing anything significant requires considerable training, e.g. mathematics, physics, music, dance. What we see is that this training is never enough — that those who are truly creative are able to go beyond their training, to let go of their training. When you prepare a speech, you prepare a particular entity, which acquires life when you let go of it. When you prepare yourself through arduous training, you acquire mental structures that can create such entities with relative ease. Yet, unless you let go of what these structures produce, you will never truly create. Almost all creative people have stories of how they struggled intensely to solve some problem or arrive at a certain level of performance, without succeeding — until they let go.

In an area where the mental structures created by training are not as obvious as they are in mathematics and physics, creative people can be forgetful of the importance of preparation, and emphasize only the importance of letting go. Here is the French film-maker Cedric Klapisch (maker of *L'Auberge espagnole*), in an interview that I heard recently: "The good film is the one that escapes you the most. ... [Y]ou cannot control what escapes you the most. ... I have greater confidence in spontaneity. ... I have the impression that it's I who decide, but in fact it isn't I who decide. There is something beyond me, or inside me. One must let one's unconscious speak, because it's the best judge. ... This is something that arouses fear. But, in any case, it is the only place that is interesting, the only space of creation that is interesting."

The great French mathematician Poincaré sees the process of creation more completely, perhaps because the mental pathways created by mathematical training are more clearly defined, and thus the flow of conscious work in them more easily recognized: "I turned my attention to the study of some arithmetical questions apparently without much success and without a suspicion of any connection with my preceding researches. Disgusted with my failure, I went to spend a few days at the seaside and thought of something else. One morning, walking on the bluff, the idea came to me, with [...] brevity, suddenness and immediate certainty, that the arithmetic transformations of indefinite ternary quadratic forms were identical with those of non-Euclidian geometry. ... Most striking at first is this appearance of sudden illumination, a manifest sign of long, unconscious prior work. ... There is [a] remark to be made about the conditions of this unconscious work: it is only possible, and of a certainty it is only fruitful, if it is on the one hand preceded and on the other hand followed by a period of conscious work. These sudden inspirations [...] never happen except after some days of voluntary effort which has appeared absolutely fruitless and whence nothing good seems to have come, where the way taken seems totally astray."

We tend to imagine that something as rational as mathematics must originate entirely in conscious processes. How different the truth is! Yet, even though we may find stories like Poincaré's inspiring, it is difficult for us to give our unconscious the space (or time) it needs. Conscious processes after all are easy to perceive and thus easy to control, whereas unconscious processes are difficult to perceive and thus difficult to control. Our central obsession as we emerged from animality into humanity has been control — of the environment, of other creatures, of other people, of ourselves. Not surprisingly we have become fearful of letting go. Yet let go we must if we are to create anything worthwhile. Perhaps the greatest importance of the act of creation is the possibility it offers each one of us of recognizing this truth — to create something alive we must push past the limits of the controllable, and find what we seek beyond them.

Bikram Phookun February 2021

# SCENE ZEROTH :



## WHERE DOES THY MIND WANDER TO??

DOODLE CREDITS: GARGI SINGH DOODLE CONCEPT: SHALIKA YEKKAR

#### Encryption of signals using Chaos

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Non linear dynamics is a subject that has its beginnings in the mid 1600s when it was developed to understand Kepler's Laws of planetary motion and universal gravitation. It has since developed into a subject with numerous uses in Physics, Mathematics and Computer Science. The usage of Non-Linear Dynamics in data encryption was something which was once popularised in 1990s with breakthroughs in synchronisation of chaos. We take a look into the work done then and then model their encryption circuit mathematically. We check for the robustness of the Encryption algorithm used by Cuomo and Oppenheim[4] and bring in ideas so as to improve the encryption scheme. We use Python programming to verify our ideas for improvement.

Keywords: Non-Linear Dynamics, Synchronization, Synchronization of Chaos, Cryptography, Steganography, Cuomo, Oppenheim, Chaos, Secure Communication, Mathematical modelling on Python

#### I. INTRODUCTION

Differential equations and iterated maps are two different types of dynamical systems. The main difference between the two is that differential equations describe the changes of a system continuously with respect to time whereas iterated maps describe the evolution of a system in discrete steps of time. When the derivative of a quantity (discrete or continuous) is equated to a function, both dynamical systems help us calculate changes in the given parameter (say  $x_1$ ) with respect to another (mostly taken as time). If the function they are equated to is linear in  $x_1$  then one can say that the dynamics of the system are linear in nature. If the function they are equated to is non-linear in  $x_1$  (or its derivatives), then it is said that the dynamics of the system are non-linear in nature. It is harder to analytically solve non-linear systems as they can't be broken down in a system.

 $\dot{\mathbf{x}}_1 = \mathbf{x}_2$ 

$$\dot{\mathbf{x}}_2 = -\mathbf{k}\mathbf{x}_2 - \mathbf{m}\mathbf{x}_1$$

FIG. 1. A linear System.

 $\dot{\mathbf{x}}_1 = \mathbf{x}_2$ 

#### $\dot{x}_2 = -\sin(x_1)$

FIG. 2. A non-linear System.

What we will see, after we have known enough of the basics to understand Kevin and Cuomo's circuits via the mathematical simulation, is how we can improve upon

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the methods in which the signal is hidden using some concepts from cryptography. We will also test out the robustness of the circuit via their dependencies on the parameters.

#### **II. CRYPTOGRAPHY**

The need for secure communication has been in existence for as long as people remember. I don't think its required for me to emphasise the need of privacy in today's world where everything is information based and a lot of it is transferred over various networks in order for the 'system' to work continually. This is where cryptography comes into the picture. Cryptography is the practice and study of techniques for secure communication in the presence of third parties called adversaries. More generally, cryptography is about constructing and analysing protocols that prevent third parties or the public from reading private messages [1].

Our main aim here is to send signals securely. We begin with some basic terms we are going to use in the context of cryptography:

- **Plaintext**: This signifies the message or the signal to be sent and understood by the sender, receiver and anyone else who comes across it. It's what we encrypt before we send it and what is received upon decryption.
- **Ciphertext**: This refers to the encrypted plaintext which cannot be understood (as something logical or intelligible) by anyone without decrypting it to get.
- **Encryption**: The process of converting plaintext into ciphertext.
- **Decryption**: The process of converting ciphertext into plaintext.

Stated above are loose definitions which are there just to help you understand the ideas we present (Fig 3).



FIG. 3. Basic outlook of the process of symmetric key cryptography

Numerous cryptographic techniques and methods have been developed over time. What we are interested in, is known as 'Steganography' (Fig 4).

**Steganography** is a technique that facilitates hiding of a message that is to be kept secret inside other messages [2]. Historically, senders used methods such as invisible ink, pencil marks, etc. What we are going to do here is mask a signal (our plaintext in this case) with chaos. An outside listener only hears the meaningless noise (ciphertext). This will be received at the receiving end and decrypted by using chaos again to get back the original signal. This will be understood better after some explanation regarding non-linear dynamics and chaos using Lorenz equations.



FIG. 4. Steganography used to hide the image of a cat in the image of a tree.

As for key exchange, there are many schemes which can be used, some examples being Diffie Hellman key exchange and ECDHE.

#### III. CHAOS

Chaos is aperiodic long-term behaviour in a deterministic system that exhibits sensitive dependence on initial conditions [3]. Chaotic systems provide a rich mechanism for signal design and generation, with potential applications to communications and signal processing [4]. We are going to use their typical broadband, noise like nature to mask the signal we wish to send.

A particularly useful class of chaotic systems are those that possess a self-synchronisation property [5]. This property is seen when we deal with 2 subsystems: a driving system (commonly known as the transmitter) and a response system (commonly known as receiver). For some synchronised chaotic systems (S.C.S.), the ability to synchronise is robust to perturbations [4]. One such system is the Lorenz system.

#### IV. LORENZ EQUATIONS AND STRANGE ATTRACTORS

The Lorenz Equations are:

$$\dot{x} = \sigma (y - x)$$
  
 $\dot{y} = rx - y - xz$   
 $\dot{z} = xy - bz$ 

#### FIG. 5. Lorenz Equations

The system (Fig 5) has two non linearities and it has a symmetry in (x, y). We have three equations. One in variable x, y, z respectively and they provide us with how their derivatives with respect to time vary depending on x, y, z,  $\sigma$ , r, b. Here  $\sigma$ , r, b are the parameters, which when varied give different Lorenz systems. When plotted in 3 dimensions, the Lorenz equations form an attractor.

Loosely speaking, an attractor is a set to which all neighbouring trajectories converge [3]. The subcategory of attractors which the Lorenz system forms is a strange attractor (for values of r greater than or equal to 24.06).

An attractor is called *strange* if it has a fractal structure. This is often the case when the dynamics on it are chaotic, but strange non-chaotic attractors also exist. If a strange attractor is chaotic, exhibiting sensitive dependence on initial conditions, then any two arbitrarily close alternative initial points on the attractor, after any of various numbers of iterations, will lead to points that are arbitrarily far apart (subject to the confines of the attractor), and after any of various other numbers of iterations will lead to points that are arbitrarily close together [6].

#### V. CUOMO AND OPPENHEIM'S CIRCUIT AND SYNCHRONISATION

Chaotic systems would seem to be dynamical systems that defy synchronisation. Two identical autonomous chaotic systems started at nearly the same initial points in phase space have trajectories which become quickly uncorrelated, even though each map out the same attractor in phase space [5]. Yet Pecora and Carroll were able to synchronize chaotic systems by linking them with common signals. Using their work, Cuomo and Oppenheim were able to design electronical circuits (a transmitter and a receiver) which would synchronise. To be precise, the receiver asymptotically approaches perfect synchrony with the transmitter, starting from any initial conditions.

Their circuit (Fig 6 and V) is based on the Lorenz equations. What they use are voltages (u, v, w) at different points of a circuit which are proportional to x, y, z in Lorenz circuits.



FIG. 6. Transmitting circuit



FIG. 7. Receiving circuit

There were 2 circuits involved, a driving circuit (x, y,

z) and a receiving circuit (xr, yr, zr). Their equations being:

$$\dot{x} = \sigma (y - x) \qquad x_r = x \dot{y} = rx - y - 20xz \qquad \dot{y}_r = rx_r - y_r - 20x_rz_r \dot{z} = xy - bz \qquad \dot{z}_r = 5x_ry_r - bz_r$$

FIG. 8. Equations of the driving and receiving circuits

As you can see, the x variable of the receiving circuit is just the x variable from the transmitting circuit and it is used to power the receiving circuit and the y and z variables of the receiving circuit have dependencies on newer values of the receiving circuit.

What we use in our mathematical simulation of the circuit are the same equations removing the numerical constants used here (so essentially the Lorenz equations themselves). What we observe is that the circuits synchronize for all initial conditions. This can be understood as follows:

$$\begin{aligned} d(t) &= (x, y, z) \\ r(t) &= (x_r, y_r, z_r) \\ e(t) &= d(t) - r(t) = \text{error signal} \longrightarrow 0 \text{ as } t \longrightarrow \infty \end{aligned}$$





FIG. 10. Error vs Time graph denoting sync after initial transition



FIG. 11. Error vs Time graph denoting sync after initial transition in 3D  $\,$ 

The 3D plot of the variables when looked at (for time t=95) looks like (Fig 15):

#### VI. USING THE CIRCUIT TO SEND HIDDEN MESSAGES

It may not seem like it but the mathematical circuit presented above can be used to send signals (with a reasonable amount of security) but it can be very efficiently be used to do so. The method used is known as steganography.

What we basically do is mask our signal under the chaos so that it won't be understandable to an intruder. The method utilised to perform this can be explained as follows:

- You generate 3 arrays of variations of variables x, y, z over time using the Lorenz equations.
- While you input the newly generated value of (say) x variable at time (t+dt) you add the value of the signal at that instant in the array.
- This happens for some time till the signal to be sent is completely added to the used variable.
- The signal array now consists of lists of values of x, y, z which don't seem like particularly useful signals on their own.
- They are received on the receiver's end and the receiver (who is aware of the parameters of the Lorenz equations used to generate the chaos masking the plaintext signal) generates the original chaos in the 3 variables.
- He/she subtracts them from the respective signal values received from the transmitter end.

• What is left, when plotted against time should give you the hidden signal, given the 2 circuits used are in sync (which they were).







FIG. 14.



FIG. 15. When seen as an audio signal or separately (x, y, z), the signal doesn't seem to form a pattern as it is chaotic in nature.



FIG. 16. The circuit on the receiving end can be plotted as shown.

#### VII. ROBUSTNESS OF THE CIRCUIT

Our encryption scheme is based on the fact that the third-party intruder must have access to the exact values of out parameters  $\sigma$ , r, b to generate a Lorenz attractor which upon subtraction would give the masked signal. So, there are a lot of values possible for our parameters  $\sigma$ , r, b to choose from. But what if the intruder had values close enough to the original values? That is why we check for the robustness of our circuit by varying parameters slightly and checking if that would give back a good-enough decrypted signal.



FIG. 17. Superimposition of signals involved





FIG. 21.





FIG. 23. Signal squared vs t at the same parameter values of the driving and receiving circuit



FIG. 24. Signal squared vs t at  $\Delta$  r,  $\Delta$   $\beta$  being 0.01, 0.1, 1 in the two circuits

Upon varying parameters r, b one can see that it is very hard to get back the original signal with a proper accuracy. The closest you can get to the actual signal is at a difference of at least 1/100 of the right value. That too at very small values of dt (time interval for the algorithm involved).

If we're worried about the fact that there is a case that the positive and negative deviations of the subtracted signal may give out something that is resembling the decrypted plaintext, we carried out a small analysis where upon using the exact same values of the parameters the decrypted signal squared looks something like Fig. 23. And upon varying  $\Delta$  r,  $\Delta \beta$  being 0.01 (blue), 0.1 (orange), 1 (green) in Fig. 24.

#### VIII. IMPROVEMENTS OVER THE ORIGINAL ENCRYPTION AND THEIR IMPLEMENTATION

**Periodic Variable switching** One can very easily encrypt the given signal on a variable. But that's not the only method possible (or one can say not the only key). If one wishes, they can split the signal between the 3 variables and then recombine it on the receiver's end by plotting the error (subtracted signals in x, y, z) on top of each other. There are many ways to achieve this as well. You basically have infinite possibilities in ways you can split the signal to be sent. One of them is splitting it in equal thirds between the variables x, y, z. The retrieved signal will look something like this (Fig. 25):



FIG. 25. The circuit on the receiving end can be plotted as shown.

One can see that such splitting of the signal induces some error over the retrieved signal and that can be improved upon by having the transmitter and receiver circuits start at the same initial conditions and to reduce error even further you can reduce the time step (which will be compensated over in execution time). Here are some instances of different cases of dividing the signal into equal thirds.

What one can observe from the cases shown is that the retrieval of the sent signal is most accurate in the variable x. That is no surprise considering that the x variable is already synchronised as the circuit starts. Keeping this in mind along with the fact that the transmission sent across after encryption (ciphertext) is basically a group of 3 arrays which when plotted in 3 dimensions give a (Lorentz) strange attractor (fractal in nature). Supposing the 3rd party intruder is able to understand and plot the 3 arrays in the x, y, z plane and see that it's a figure looking like this and knows the fact that we requires the exact parameters of the Lorenz's equations to recreate the original attractor and subtract the values of the corresponding parts of the 3 arrays. Now all the intruder requires is to know the points as which the sending of the signal begins to have (assuming they have the 3 parameters as well) his recreated signal synchronised and subtracted with the original transmission to get back the sent signal but that would still be hard to comprehend considering you don't have any patterns which you can see in the chaotic attractor to figure that out. What that basically means is that it'll be harder to find the signal masked under chaos when split into parts and stored in different variables than a single variable as you can have the transmitted attractor's shape distorted only ever so slightly to make it seem like a transmission which wasn't carrying any signal at all.

Curvature based variable switching One of the most efficient methods in parameter switching would be continuous parameter switching. Granted that this will have a significantly larger amount of error as the switching takes place but this would be the one which the intruders least suspect of having a signal contained in it. A logical way to proceed in this direction would be curvature-based switching of the parameters carrying the signal. What we mean by that is that at points (on the Lorenz attractor) whose curvature in 3 dimensions is the most (depending on three classes of curvature which can be easily decided later), you use the x variable to mask the signal under the chaos. In cases (the predetermined class) where the curvature is least, you use the z variable to mask the signal under the chaos. In all other cases you use the y variable. This scheme has been created after studying the error dependencies on parameters present in the non linear equations governing the dynamics of the 3 variables.

#### IX. CONCLUSION

To conclude, we have mathematically modelled the circuit created by Cuomo and Oppenheim based on the Lorenz equations. The circuit, although was an integral part for the enabling of the encryption in real life but considering it was made approximately 28 years ago, our efforts were in the direction of understanding and improving the encryption used as the circuit's basis. We were able to take a look into the working of the circuit and reproduce the results via programming the logic using Python. We have used the program to check the robustness of the algorithm of encryption and were able to produce usable results. Our approach towards improving the the encryption algorithm was discussed and how that may help this unconventional mode of data encryption strong enough to challenge modern forms of third party attacks and hopefully attacks by a quantum computer. This can be extended to higher number of variables and thus improving our encryption abilities.

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[6] URL: https://en.wikipedia.org/wiki/Attractor

The whole research project can be found at the author's research-gate account https://www. researchgate.net/profile/Chaitanya-Varma

# SCENE 1:



## THE PHYSICIST'S LOVE LIFE

DOODLE CREDITS: GARGI SINGH DOODLE CONCEPT: SWAPNILA CHAKRABARTY

#### Simulation of Lennard-Jones Potential on a Quantum Computer

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Simulation of time dynamical physical problems has been a challenge for classical computers due to their time-complexity. To demonstrate the dominance of quantum computers over classical computers in this regime, here we simulate a semi-empirical model where two neutral particles interact through Lennard-Jones potential in a one dimensional system. We implement the above scenario on IBM quantum experience platform using a 5-qubit real device. We construct the Hamiltonian and then efficiently map it to quantum operators onto quantum gates using the time-evolutionary unitary matrix obtained from the Hamiltonian. We verify the results collected from the qasm-simulator and compare it with that of the 5-qubit real chip ibmqx2.

Keywords: Quantum Simulation, Time-Complexity, IBM Quantum Experience, Quantum Gates, Schrødinger's Equation

#### I. INTRODUCTION

The age of quantum simulation on quantum computers started in the 1980s when Benioff showed that a computer could operate under the laws of quantum mechanics, which is further backed with the famous paper by Feynman at the first conference on the physics of computation, held at MIT in May 1980. He presented in the talk, titled as 'Quantum mechanical Hamiltonian models of discrete processes that erase their histories: application to Turing machines', how efficient can a quantum computer be for NP Hard and EXP problems [1] and there proposed a basic model for a quantum computer[2]. The problem of simulating the full-time evolution of arbitrary quantum systems is intractable due to the exponential growth of the size of a system. For example, it would need  $2^{47}$  bits to record a state of 47 spin- $\frac{1}{2}$  particles, which is equal to  $1.4 \times 10^{14}$  bits, a little less than 20 TB which is the largest storage capacity of hard disk currently [3-5]. Feynman gave simple examples of one quantum system simulating another and conjectured that there existed a class of universal quantum simulators capable of simulating any quantum system that evolved according to local interactions [6]. These local interactions, if controlled, could be exploited to mimic the dynamics of the quantum systems very efficiently. Essentially, each tweaking will move the system in specific directions in Hilbert space which corresponds to an operation, we are mimicking through it.

Classical information was first carefully defined by Shannon in his paper in [7]. Any change could be considered as a signal if it can be encoded with some data by the sender and retrieved by the receiver. 'Bit' is the most popular unit of classical data. A bit can either store 1 or 0 in form of voltages generally. However, in simple

terms, due to the quantum nature of the qubit (quantum bit), it can store more information than a bit. A qubit is an abstraction of any two-state quantum system, like a bit, is an abstraction of a two-state classical system. Information can be encoded on photons, spins, atoms, or the energy state of electrons in quantum dots. A gate is the function of bits. Since a qubit is more versatile than a bit (can be in a state, which is a combination of its two states simultaneously, is one of them), there are many more fundamental gates. These gates control the output and a series of them can perform any complicated tasks. Though now the number of qubits required will be 47 (exponentially small), the number of operations is still the same to evolve the system through time. It is a branch of computer science, physics, and engineering in its infant stages, growing rapidly. The quest for more QS is described in referred works [8-22]. Currently, using an online accessible quantum computing platform, IBM quantum experience, a number of research works have been performed among which the following [23-47] can be referred.

The organization of the paper is as follows. In Section II, the theoretical background and the algorithm is laid out. Then the mapping is given in Sec. III. The simulation is then done using the Qiskit of IBM quantum experience platform. Finally, the experimental results of simulation and comparisons are given in Sec. IV. The analysis and future scope is discussed in the Sec. V.

#### **II. SIMULATION**

Since on a computer, the notion of infinitesimal change can not be applied. Thus, as a numerical approximation, the system's evolution has been studied discretely (Fig. 1). We can always make our time step sufficiently small, for depicting it as close as to a physical system. But there is always a limit to this. This limit depends on the ability of our computer to handle these numbers, the

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runtime one can afford and the precision one wants in the system. Therefore, only the discretized system and its operators are being considered in subsequent sections.



FIG. 1. The system will evolve through time into discrete time stages. Apart from the discontinuity in time, we have discrete spatial points too. As long as two black boxes give out the same output to the same input, we regard them as identical. This is the philosophical idea behind simulations. Though the mechanism could be different in both the cases, real-world and computer, we can consider them to be same machinery.

It is said that, given the dynamical law (Eq. 2) of a system, the whole state-space can be determined for given initial conditions[48]. To mimic this, the digital quantum simulations work on the following main steps: initial state preparation, time evolution, and measurement. Consequently, we designed our simulation based on that principle.



FIG. 2. We can construct perfect laws if we have all the inputs and their corresponding outputs. Given the information in this black box, you can predict the future state of your system depending upon the current state.

The main algorithm is focused on designing this time evolution of the system since that defines the system, and the other two parts are computational. Quantum systems evolve according to a unitary operator which is given as  $U(t) = e^{-\iota \hat{H} t/\hbar}$  [49]. The  $\hat{H}$  is called the Hamiltonian of the system and is dictated by the components of the system. This unitary operation can be split into free and interacting operators using Trotter's formula [50].

$$exp(-\iota\hat{H}t) = exp(-\iota(\hat{V} + \hat{K})t)$$
$$= exp(-\iota\hat{V}t)exp(-\iota\hat{K}t)exp(\mathcal{O}(\Delta t^{2}))$$

We have used first order truncation but higher order approximation methods have also been developed [51-53] that gives more accurate results at the cost of more number of quantum gates per time step.

#### A. Hamiltonian

The time-dependent Schrodinger wave equation in one dimension is

$$\iota\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi \tag{1}$$

where  $\Psi$  is the time dependent wave-function that contain all the information about a system and  $\hat{H}$  is the Hamiltonian of the system. The general form of Hamiltonian contains the operations associated with the kinetic and potential energies. Hence, for a wavefunction of two particles of mass  $m_1$  and  $m_2$  in one dimension can be written as,

$$\hat{H} = \frac{-\hbar^2}{2m_1} \frac{\partial^2}{\partial x_1^2} + \frac{-\hbar^2}{2m_2} \frac{\partial^2}{\partial x_2^2} + V(x)$$
(2)

Here,  $\iota \hbar \frac{\partial}{\partial x}$  is the momentum operator for a wavefunction. The particles will be interacting through a short range force defined by Lennard-Jones potential [54-31] which is given by

$$V(x) = \epsilon \left[ \left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^6 \right]$$
(3)

where  $\epsilon$  and  $\sigma$  are constants for the particles in consideration and x is the relative distance between them. To get rid of the unnecessary clutter, we took  $\epsilon = 1/4$ and  $\sigma = 1$  and  $m_1 = m_2 = 1/2$ . This potential has two parts. The negative part has a larger range and responsible for the attraction, and the positive part has a shorter range and is responsible for the repulsion. It is widely used as a model for inter-molecular interaction which results in a type of ideal fluids called Lennard Jones fluid [62]. Therefore with this information, our Hamiltonian becomes (in abstract form)

$$\widehat{H} = \widehat{p}_1^2 + \widehat{p}_2^2 + (\widehat{x}^{-12} - \widehat{x}^{-6}) \tag{4}$$

where  $\hat{p}$  and  $\hat{x}$  are the momentum and position operators respectively.

#### B. Discretization

Since, the state-space is 'discretized' for simulation, the Hamiltonian operator, therefore, is given as

$$\widehat{H}_d = \widehat{p}_{1d}^2 + \widehat{p}_{2d}^2 + (\widehat{x}_d^{-12} - \widehat{x}_d^{-6}) \tag{5}$$

where  $\hat{p}_d$  and  $\hat{x}_d$  are the momentum and position operators in finite dimensions. These position and momentum operators can be expressed as in their diagonalized form which has eigenvalues as their diagonal entries in their respective space.



FIG. 3. Lennard-Jones potential is a short range potential which is one of the best suited models for showing van der Waals interaction. As a result, it has become fundamental in modelling the molecular interaction of real substance. The points highlighted here are the mid points of region we took for discretization. The distance corresponding to the minimum potential from the origin is at x = 1.1225 and generally is called bond length for molecules.

#### 1. Position operator

If we take N distinct lattice points of our one dimensional discrete space, there could be  $N^2$  possible position-eigenstates for this system, from the permutations. Therefore, 4 discrete spatial points have been taken wherein they can exist. This choice of size of lattice could be any, but depending on the increasing complexity, the optimum limit is to be taken. The resolution chosen is coarse due to the technological limitations of the current quantum computers in handling a certain number of qubits. After choosing an origin we can give integral values to those positions, with respect to our length scale. As discussed before, there would be 16 eigenstate vectors representing the spatial permutations of particles (Fig 4).

If  $X_i$  and  $X_j$  are the distances from the origin of the particle, the relative distance could be written as,

$$X_{ij} = |X_i - X_j| = |i - j|\Delta l \tag{6}$$

where  $\Delta l$  is resolution of space. Now, when i = j,  $X_{ij} \in (0, \Delta l)$ , which means the particle could be anywhere between  $X_i + \frac{\Delta l}{2}$  and  $X_i - \frac{\Delta l}{2}$ , if other particle is fixed and vice-versa. The most suitable approximation would be to take  $X_{ij} = \langle \Delta X \rangle = \frac{\Delta l}{2} = \frac{1}{2}$ .



FIG. 4. Spatial grid: from the picture of any edge of this dotted grid as our spatial grid we can understand this state-space contains all these permutations. The potential is computed on the mid-points of each section of one-dimensional space. This is to take into account that the particle could be anywhere within a section. For instance, if we assume the origin to be at the bottom left corner, the first row tells about the states when one particle is at the origin and the other particle can occupy any of the four spatial positions and thus the state vector corresponding to that case is encoded.

Therefore, the potential operator in diagonal form becomes, given the position eigenvalues

$$\widehat{V} = Diag\left\{ \left( |i - j|\Delta l \right)^{-12} - \left( |i - j|\Delta l \right)^{-6} \right\}$$
(7)

and  $i \neq j$  for this notation and  $i, j \in \{0, 1, 2, 3\}$ .

#### 2. Kinetic Energy Operator

The kinetic energy of a particle (having mass equals to 1/2 for simplicity) is related to momentum as  $\hat{K} = \hat{p}^2$ . Since to use the diagonalized form, we first write the wave function in the momentum representation. The momentum eigenvalues of a particle in P-basis for its eigenstate  $|l\rangle$  is given by [63],

$$p_l = \begin{cases} \frac{2\pi}{2^n}l & 0 \le l \le 2^{n-1}\\ \frac{2\pi}{2^n}(2^{n-1}-l) & 2^{n-1} < l < 2^n \end{cases}$$
(8)

where n is the dimension of the space. Since for each individual particle there are 4 possible states in position space, there will be 4 states in momentum space too.

The 'total' wavefunction for two independent free particles can be decomposed according to the  $\Psi = \Psi_1 \Psi_2$ , where  $\Psi_i$  is the wavefunction of an individual particle [64]. Therefore, for each particle the kinetic energy operator is

$$\widehat{K}_{l} = \left(Diag\left\{0, \frac{\pi}{2}, 0, -\frac{\pi}{2}\right\}\right)^{2} \tag{9}$$

But before operating directly to the wavefunction, we need to transform our state, using quantum Fourier transform, into momentum basis representation.

#### 3. Quantum Fourier Transform

Quantum Fourier transform is a discrete Fourier transform on complex amplitudes of a quantum state. On applying to our wavefunction in X-basis, we can transform it to P-basis, so that the diagonalized momentum operator can be applied to the respective eigenfunctions in p-space. An inverse quantum Fourier transform can be done to bring back the wave function into position-space. It is a unitary operator so can be physically realizable. The general form of QFT can be given as [65],

$$|\Phi\rangle = (QFT) |\Psi\rangle = (QFT) \sum_{n=0}^{N-1} c_n |n\rangle = \sum_{k=0}^{N-1} b_k |k\rangle$$
(10)

where

$$b_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} c_n exp\left(\frac{2\pi \iota nk}{N}\right)$$

where N is the total number of bases. Therefore the momentum operation can be applied as follows

$$\hat{P}_x |\Psi\rangle = (QFT)^{-1} \hat{P}_l (QFT) |\Psi\rangle \tag{11}$$

where  $\hat{P}_x$  is momentum operator in X-basis, (QFT), (QFT)<sup>-1</sup> refers to quantum Fourier transform operators and  $\hat{P}_l$  is the momentum operator in P-basis.

The unitary matrix of QFT is given by [66],

$$U_{QFT} = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} e^{2\pi \iota \frac{xy}{N}} |P\rangle \langle X| \qquad (12)$$

#### C. Wavefunction

After using the first order approximated Baker-Campbell-Hausdorff formula, the final unitary operation on the wave function for our system looks like as follows, and thus the time evolution can be studied.

$$\hat{U} = e^{-\iota \hat{H} \Delta t} \approx U_{(QFT)} (e^{-\iota \hat{P}^2 \Delta t}) U_{(QFT)}^{-1} (e^{-\iota \hat{V} \Delta t}) \quad (13)$$

Also, any arbitrary wavefunction can be decomposed through their eigenfunctions in their respective space.

$$|\Psi(x_1, x_2, t)\rangle = \sum_{i,j=0}^{2^n - 1} \psi(x_i, x_j, t) |ij\rangle$$
 (14)

where  $\sum_{i,j=0}^{2^n-1} |\psi(x_i, x_j, t)|^2 = 1$  and  $|ij\rangle$  is the twodimensional kronecker delta function. Therefore, as a summary, the evolution of state is given using Eqs. 13 and 14.

$$\sum_{i,j=0}^{15} \psi(x_1, x_2, t + \Delta t) |ij\rangle = \hat{U} \Big[ \sum_{i,j=0}^{15} \psi(x_1, x_2, t) |ij\rangle \Big]$$
(15)

These states and operators are now mapped onto qubits and qubit-gates respectively for a simulation.

#### **III. CIRCUIT IMPLEMENTATION**

#### 1. Main Idea

One of the postulates of quantum mechanics says that every quantum observable (like momentum, position, and energy) is associated with an operator, whose eigenvalues correspond to all the possible values of that observable. In a certain sense, this incorporates the physical indeterminacy with the measurement of a quantum system. Every mathematical operation and so does any physical operation can be boiled down to fundamental logic gates. Any quantum logic gate for a two-state system is realized using four fundamental gates called Pauli gates. So our dynamical system can be mapped to an electronic device by mere quantum gates [67]. Therefore, on the basis of operations, we design the combination of gates that further acts on the set of qubits on which the wavefunction is mapped. The phase rotation is carried out by a series of CU1 gates (controlled-U1) and Hadamard gates [68].

Since we are simulating 4 lattice points that combine 16 dimensional Hilbert space, therefore we need 4 qubits to describe those 16 different states and only 1 more gubit is used as an ancillary qubit. Further, we optimize the circuit by reducing the redundant gates and exploiting the symmetry showed by the operators. We manage to apply the kinetic energy operator without any use of an ancillary qubit. Following this algorithm, one would need N+1 qubits to implement potential with most efficiency [69,70] where  $2^N (N \in \mathbb{N})$  is the number of lattice points. Other values of lattice points would result in wastage of qubits since n qubits can represent up to  $2^N$  states. These  $2^N$  qubits that together store the complete state of the system (Eq. (14)). The probability distribution of each qubit state corresponds one to one with the probability distribution of each eigenfunction after the time evolution (Eq. (16)).

The final state in spectral decomposed form will be interpreted as

$$|\Psi\rangle \to \sum_{l=0}^{2^4-1} \chi \,|l_b\rangle \tag{16}$$

where  $\chi$  is the amplitude of respective eigenstates of the system which is cloned by the qubit's state  $|l_b\rangle$  expressed in binary.

#### 2. Technique

The circuit has been coded on Qiskit platform [71], for which the main idea is presented, through graphical circuit composer, here. The complete circuit can be segregated into 5 parts as suggested by Eq. (13).

- initial state preparation
- potential energy operation
- kinetic energy operation
- quantum Fourier transform operation
- measurement operation

As mentioned, each state of qubits will encode an initial state of the system. The IBM quantum experience default initial state is  $|000..\rangle$ . To obtain various initial conditions, we can use NOT gates or simply Hadamard gates.

For designing the circuit for the potential operator (Fig. 6), we took the advantage of the symmetry in the arguments (Table I) and the fact that phase rotation works only on  $|1\rangle$  state. For that, we used a series of Toffoli gates (two control NOT gate) to invert the remaining input and apply cU1 gates for a particular state using filtering technique. This technique picks a particular state and applies a phase-rotation to that state according to the unitary operator given (Fig. 5). The final circuit has been optimized by canceling redundant gates (Fig. 6 and Fig. 7). The real code includes more iterations and a smaller time-step but the figure here was given just to give an idea.

| States         | Diagonal Entries of Potential Energy operator |                                  |         |
|----------------|---|----------------------------------|---------|
|                |   | after extraction of common phase |         |
|                |   |                                  |         |
| $ 0000\rangle$ | 0.49024                                       | 0                                | $V_1$   |
| $ 0001\rangle$ | -0.25   | -0.74024                         | $V_2$   |
| $ 0010\rangle$ | -0.16711175                                   | -0.65735175                      | $V_3$   |
| $ 0011\rangle$ | 0901352                                       | -0.5803752                       | $V_4$   |
| $ 0100\rangle$ | -0.25   | -0.74024                         | $V_2$   |
| $ 0101\rangle$ | 0.49024                                       | 0                                | $V_1$   |
| $ 0110\rangle$ | -0.25   | -0.74024                         | $V_2$   |
| $ 0111\rangle$ | -0.16711175                                   | -0.65735175                      | $V_3$   |
| <u> </u>       |   |                                  |         |
| 1000           | 0 16711175                                    | 0.65795175                       | IZ.     |
| 1000           | -0.10711175                                   | -0.03733173                      | V3<br>V |
| 1001           | -0.25   | -0.74024                         | $V_2$   |
| 1010           | 0.49024                                       | 0 74094                          |         |
| 11100          | -0.20   | -0.74024                         |         |
| $ 1100\rangle$ | 0901332                                       | -0.0800702                       |         |
| $ 1101\rangle$ | -0.10/111/5                                   | -0.05/351/5                      |         |
| $ 1110\rangle$ | -0.25   | -0.74024                         |         |
|                | 0.49024                                       | 0                                | $V_1$   |
|                |   |                                  |         |

TABLE I. Diagonal entries of potential operator. Since the probability amplitudes are independent of the global phase, we can omit it in the expression which heavily simplifies our equations. This does not affect the relative phase of each operator. The symmetry shown, can be exploited to optimize our circuits.



FIG. 5. The filtering has been explained here with an example. The main purpose of the circuit is to operate certain operations on the  $|1\rangle$  depending upon the function it is performing. This circuit picks out the state  $|001\rangle$  with the help of Toffoli gates, then the phase-rotation has been applied to any of the qubits having state  $|1\rangle$ . The mirror circuit succeeding, is to revert back the changes applied for filtering which is so that other operations could be applied. In the simulation we optimized it so as to reduce the ancillary qubits.

The similar technique applies for designing momentum operator (Eq. (9)). Since, it is applied to each pair of the qubits separately, it is mapped onto  $R_z$  and CPhase



FIG. 6. Graphical circuit for potential operator. The circuit is divided as given in the Section III 2. The circuit is exploiting the symmetry of potential operator. The first three CNOT gates invert the three inputs which is equivalent of folding half of the table I coinciding it to the other half. The rest of the circuit is filtering and applying phase rotation corresponding to the state.



FIG. 7. Graphical circuit for momentum operator, Qft. The circuit is divided as given in the Section III 2. As we go from right to left, we prepare an initial state, apply quantum Fourier transform, then act upon by the momentum operator, after which to revert back the changes inverse quantum Fourier transform is applied.

gates which are operated individually on  $|q_1q_0\rangle$  and  $|q_3q_2\rangle$  (Fig. 7).



FIG. 8. General n-qubit QFT circuit where H stands for Hadamard gate and R stands for the phase-rotation gate for customizing input argument.

The standard circuit implementing the QFT to nqubits is executed through a series of Hadamard and cU1 gates (controlled-phase gates) given in Fig. 8[72]. The argument of each individual cU1 gate is given by,

$$\phi = \frac{2\pi}{2^N}\iota\tag{17}$$

N being the number of qubits. The circuit for inverse QFT is mirror image of QFT with conjugate arguments (i.e.,  $-\phi$ ) (Fig. 9).

For 2-qubit system, our circuit is represented on IBM circuit composer as given in Fig. 9,



FIG. 9. The circuit implementation of QFT and QFT inverse on IBM circuit composer for 2-qubit system.

| States       | Diagonal Entries  |                                      |
|--------------|-------------------|--------------------------------------|
|              | $ K\rangle$       | $e^{-\iota K\rangle\Delta t}$        |
| $ 00\rangle$ | 0                 | 1                                    |
| $ 01\rangle$ | $\frac{\pi^2}{4}$ | $e^{-\iota \frac{\pi^2}{4}\Delta t}$ |
| $ 10\rangle$ | 0                 | 1                                    |
| $ 11\rangle$ | $\frac{\pi^2}{4}$ | $e^{-\iota \frac{\pi^2}{4}\Delta t}$ |

TABLE II. Phase values of Kinetic Energy operator

The last step is the measurement, where the quantum state is measured and stored in a classical register.



FIG. 10. The final state of qubits is stored in a classical computer through registers. We need to store amplitude of each state. Since, an amplitude is a complex number, we need a space equivalent to storing 2 float numbers. Using 32 bits (4 bytes) for each floating point number, a quantum state of n = 4 qubits require 128 byte of memory which rise upto 1GByte for n=27.

The  $\Delta t$  (time step) is taken to be 0.05 and 0.01 and convergence is checked. The final time is chosen to be  $\sqrt{2}$  units calculated from the constants  $\sigma, \epsilon$  and m.

#### IV. EXPERIMENTAL RESULTS

In order to perform the simulation, various time-steps have been taken with different initial conditions to study the system. The states  $|0001\rangle$ ,  $|0100\rangle$ ,  $|0110\rangle$ ,  $|1001\rangle$ ,  $|1011\rangle$ , and  $|1110\rangle$ are behaviourally similar points and correspond to classical minimum energy. The evolution of  $|0001\rangle$  has presented here in Fig. 13 and Fig. 14, upto a natural time scale of the system. The state being classically most favourable, diffuses to other states having peaks near the states corresponding to the same classical least energy. To compare the simulated result from "IBM Q QASM simulator", a similar simulation is performed on quantum processor "ibmqx2" [73] which is depicted in Fig. 15. In both the cases, the number of shots taken were 8192. In Fig. 11, the final state after  $29^{th}$  iteration is obtained from Qiskit platform, and though this single snapshot is insufficient to capture the whole process, it can be seen that the wavefunction is slowly diffusing out to other states showing increasing position uncertainty with time. To see the resemblance, we simulated the states  $|0001\rangle$ ,  $|0100\rangle$ ,  $|0110\rangle$  and  $|1110\rangle$  and have shown the wavefunction after  $29^{th}$  iteration in Fig. 16. Similar behavior is obtained, which verifies the symmetry of the space. In contrast to this, we can see for the state  $|0011\rangle$ , the wavefunction diffused to other states rather quickly in Fig. 17. Snapshots in an irregular range are given, which finally gives the wave function a form with a maximum probability in the same classical energy-like states.

Ultimately, this simulation can be generalized for arbitrary spatial dimensions and the number of interacting particles. The simulation can also be carried out using an arbitrarily large number of qubits to make it more precise. The same type of optimization can be done by exploiting the symmetries of eigenvectors.



FIG. 11. This is the enlarged image of the  $29^{th}$  evolution of the state  $|0101\rangle$ . The result is obtained through the IBM Q5 Yorktown, backened name: ibmqx2 and version: 2.2.5.

#### V. DISCUSSION AND CONCLUSION

We successfully simulated a two-particle system in a single-dimensional grid using short-range interaction. We achieved a recognizable behavior of the interaction within the framework of 5 qubits. The results shown in Fig. 15 indicates that simulations are not appropriate for closed systems because of the characteristically different results. Since qubits are in noisy environment, we would simulate Hamiltonian which are not coherence preserving [74]. Although the results achieved are radically (Fig. 16, Fig. 11) different than one would expect them to be, classically, which implies the need for a more refined grid. As the characteristic behavior is only noticeable in the short range of particles, we can increase the grid size without increasing the interactions. Similarly, we would like to simulate this in 3-dimensional space using 3-D meshing techniques. The accuracy of results will depend on the time step whose suitable value needs to be computed [6]. These amendments can only be made once we develop ways to process more qubits. After that, we hypothesize that we can simulate multibody quantum systems and delve deeper into the statistical nature of quantum systems.



FIG. 12. Scanning this QR code redirects to the author's research-gate page where you can find the entire research project alongside acknowledgements and bibliography



FIG. 13. Time evolution of the state  $|0001\rangle$  which corresponds to classically least energy state. From the number of iterations one can infer that it is relatively stable state. The evolution is presented upto  $9^{th}$  iterations in this diagram.



FIG. 14. The previous evolution is continued in this diagram. The evolution of each position eigenvector from  $9^{th}$  to  $28^{th}$  iteration is presented after which the states are diffused in other states comparably. The last state is again presented in Fig. 15.

Comparison between ibmqx2 and qasm simulator



FIG. 15. Experimentally obtained results from "ibmqx2" for probability distribution. Similar observations have been obtained as case "Qasm-Simulator". The horizontal axis represents the distinct states and the vertical axis represents the corresponding probabilities. The orange bars corresponds to "Qasm-Simulator" and the blue bars corresponds to "Ibmqx2".



FIG. 16. Final state after  $29^{th}$  iterations for different initial states showing similar behaviour is given. The results were obtained by Qasm-Simulator. The various states are  $|0001\rangle$ ,  $|0100\rangle$ ,  $|0110\rangle$  and  $|1110\rangle$  respectively.



FIG. 17. The following are snapshots of discrete time steps of evolution of state  $|0011\rangle$ . The snapshots are for  $1^{st}$ ,  $3^{rd}$ ,  $5^{th}$ ,  $7^{th}$ ,  $13^{th}$ ,  $16^{th}$  and  $20^{th}$  time-step. You can see the pattern emerging which gives the higher probability to other states with same classical energy. Apart from this you can see that it diffused to other states at a higher rate than other initial condition.
# SCENE 2:



# METAHUMAN PHYSICS TRICKS

DOODLE CREDITS: DITHAINGAM PANMEI

# Dynamics of crime: A review

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# PRELUDE

Crime is a complex social phenomenon, experienced in day-to-day life at various levels. There exists no exact definition of crime but over time, sociologists have come to a consensus on some defining factors. It is usually described as unlawful activities or behaviour that violates social norms. The study of crime and its causes, i.e., criminology is a pluri-disciplinary field. Throughout history, different schools of thought have proposed different and sometimes conflicting ways of considering crime. In this paper, we are interested in the possible contributions of mathematical modelling in analytically understanding crime.

Mathematical modelling of crime. It makes us wonder how even such a socially dynamic problem can be accounted for using mathematics. Many attempts have been made to model the spread of crime using different approaches and parameters. We follow a couple of such notable works in mathematical modelling of crime and make an attempt at understanding how an intangible social issue with multiple dimensions to look at, can be quantified and systematically studied.

The first paper is a simple two compartment model that considers that crime spreads through only interactions. The model is simple yet very effective in pointing out non-trivial aspects in the spread and control of crime which cannot be unveiled with statistical analysis. The second paper includes more compartments as it takes punishment and repeat offenders into account. It is more realistic and considers that crime can spread even without interactions. The analysis provides us insights into how effectively punishment can curb crime and its consequences which can employed in making policies accordingly.

These multi-dimensional complex non-linear models can be mathematically challenging and tedious to analyse. Therefore, we have skipped most of the rigorous maths involved. Those who are interested in learning more can go through the reference for a detailed understanding of the mathematics and explore more such models [?][?].

### I. TWO-COMPARTMENT MODEL

### Introduction

This model presents a simple approach to mathematically model crime in an artificially simulated society that has been broadly divided into two groups - Criminal Minded  $(C_p)$  and Non-Criminal Minded  $(N_p)$ . It uses the well-known prey-predator interaction system to model the dynamics between the two sets of population.

The nonlinear differential equations take into consideration the logistic growth of the non-criminal population, decay rate which is linearly proportional to the criminal population, change in both the populations due to interaction and a law enforcement factor which implies that the criminal population is being removed through some means. This model has been discussed in detail in the following sections.

# Mathematical model

The dynamics of the two classes of population is given by the following set of nonlinear differential equations:

$$\frac{dN_p}{dt} = \mu N_p \left( 1 - \frac{N_p}{K} \right) - aN_p C_p$$

$$\frac{dC_p}{dt} = -\gamma C_p + aN_p C_p - l_c C_p$$
(1)

with  $N_p(0)>0$  and  $C_p(0)>0$  as the initial sizes of  $N_p$  and  $C_p$ .

Description of various parameters is as follows:

- $\mu$  is the intrinsic growth rate of  $N_p$ .
- K is the carrying capacity, for the non-criminal population in the absence of criminal population which refers to the maximum possible population of noncriminals that can be achieved. Thus,  $N_p$  follows a logistic growth rather than an exponential growth.
- a is the interaction rate, i.e., the rate at which a non-criminal transforms to a criminal on interaction. This basically refers to the effectiveness of an interaction between a criminal and a non-criminal.
- $\gamma$  is the natural mortality rate of criminal population.
- $l_c$  is the coefficient of law enforcement, i.e., the measure of law enforcement.

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# Equilibria and stability analysis

To understand the qualitative behaviour of the dynamical system the equilibrium points are calculated and the stability analysis of equilibrium points of the nonlinear system is done by using the well known technique of linearization. The Jacobian in the neighbourhood of each equilibrium point is calculated, eigenvalues of the Jacobian are found and the stability of the equilibrium points is determined on the basis of sign of the eigenvalue.

Concepts of Stability theory used in analysis of this paper are as follows:

- If all the eigenvalues of the Jacobian have negative real parts, then the equilibrium point is a stable point.
- If atleast one eigenvalue of the Jacobian has a positive real part, then the equilbrium point is an unstable point.
- Types of equilibrium points:

(i) Node- When all the eigenvalues are real and are of the same sign.

(ii) Saddle- When all the eigenvalues are real and are of opposite signs.

For the model (1) the equilibrium points are as follows:

- 1. Trivial equilibrium point,  $E_1 = (0,0)$ .
- 2. Predator-free equilibrium point,  $E_2 = (K,0)$ .
- 3. Interior equilibrium point,  $E_3 = \left(\frac{\gamma+l_c}{a}, \frac{\mu}{a^2K}(aK (\gamma + l_c))\right).$

Analyzing the stability of equilibrium points using linearization technique:

1. Trivial equilibrium point,  $E_1 = (0,0)$ : The Jacobian matrix around  $E_1$  is found to be,

$$J_{E_1} = \begin{bmatrix} \mu & 0\\ 0 & -(\gamma + l_c) \end{bmatrix}$$

which has eigenvalues  $\mu$  and  $-(\gamma + l_c)$ . Therefore,  $E_1$  is a saddle point and the system would be unstable around  $E_1$ . It can be noticed that  $l_c$  doesn't play role in stability of  $E_1$ . No matter how large the law enforcement is, both the populations are bound to increase.

2. **Predator-free equilibrium point**,  $E_2 = (K,0)$ : We find the Jacobian matrix around  $E_2$  as,

$$J_{E_2} = \begin{bmatrix} -\mu & -aK\\ 0 & aK - (\gamma + l_c) \end{bmatrix}$$

which has eigenvalues  $-\mu <0$  and aK- $(\gamma + l_c)$ . So  $E_2$  would be a saddle point when aK>  $(\gamma + l_c)$ , hence the system would be unstable around  $E_2$  and  $E_2$  would be a stable point when aK<  $(\gamma + l_c)$ , hence the system would be stable around  $E_2$ . Here, it can be observed that the law enforcement plays a positive role in the stability of  $E_2$ . When  $l_c$  is larger,  $E_2$  is stable.

3. Interior equilibrium point,  $E_3 = \left(\frac{\gamma+l_c}{a}, \frac{\mu}{a^2K}(aK - (\gamma + l_c))\right)$ : We find the Jacobian matrix around  $E_3$  as,

$$J_{E_3} = \begin{bmatrix} -\frac{\mu}{aK}(\gamma + l_c) & -(\gamma + l_c) \\ \frac{\mu}{a}(aK - (\gamma + l_c)) & 0 \end{bmatrix}$$

The eigenvalues corresponding to the Jacobian matrix have complicated expressions so to simplify the analysis we calculate the trace  $(tr(J_{E_3}))$  and determinant  $(det(J_{E_3}))$  of the matrix and use the Routh-Hurwitz criterion which states that a second order system is stable if all the coefficients of the quadratic characteristic polynomial  $(\lambda^2 - tr(J_{E_3})\lambda + det(J_{E_3}))$  are positive.

We find that  $\operatorname{tr}(J_{E_3}) = -\frac{\mu}{aK}(\gamma + l_c) < 0$  and  $\det(J_{E_3}) = \frac{\mu}{a}(\gamma + l_c)(\mathrm{aK-}(\gamma + l_c))$  which is positive when  $\mathrm{aK} > (\gamma + l_c)$ . Therefore, the system is stable around  $E_3$  when the above conditions are satisfied. It can be noted that the conditions for stability of  $E_2$  and  $E_3$  are reverse. So  $l_c$  has negative effect on the stability of  $E_2$  and  $E_3$ .

### Saddle-node bifurcation

It can be noticed that, the model system (1) has three equilibrium points  $E_1$ ,  $E_2$ ,  $E_3$  when  $l_c < aK-\gamma$ , two equilbrium points  $E_1$  and  $E_2$  when  $l_c > aK-\gamma$  and when  $l_c =$  $aK-\gamma$ ,  $E_2$  and  $E_3$  coalesce hence leaving the system with two equilibrium points  $E_1$  and  $E_2$ . Also, the jacobian has a zero eigenvalue at  $l_c = aK-\gamma$ . Thus, the system undergoes saddle-node bifurcation with the bifurcation parameter being the coefficient of law enforcement  $l_c =$  $aK-\gamma$ .

### Discussion

Some special cases of this model can be considered by neglecting the logistic growth of the noncriminal population and the law enforcement. In the former case, the model will reduce to a very basic model with no constraints in terms of availability of resources or societal laws. It explores the case where the Non-criminal (Prey) and Criminal (Predator) populations grow linearly with no external bounds. This system is found to be neutrally stable where the populations oscillate periodically. It is seen that the criminal population rises with the rise of non-criminal population, however, non-criminal population falls with the rise of criminal population which is typical of a prey-predator system. The latter case by neglecting law enforcement but considering a logistic growth of noncriminal population analyses the effect of availability of resources on the system. It reveals that limited availability of resources to the non-criminal population results in rise of criminality. The three different models discussed in the paper resemble the real life scenario in different situations.

# II. SIX-COMPARTMENT MODEL

### Introduction

This paper presents a mathematical model of crime that considers crime as a social epidemic process. The underlying basis of this model is that the major reason for the spread of crime is social interaction. This model incorporates the behavioral changes in the various compartments of populations. Nobody is born as a criminal, crime is learned by the non-criminal population from the criminal population due to reasons such as lack of resources, greedy nature, social pressure, etc. The effectiveness of the interaction in the spread of crime depends on various factors such as social environment, degree of enforcement of the law, the honesty of law officials, etc. Taking the above points into consideration different classes for individuals concerning the crime and a justice system are formed. A mathematical model of epidemiological type is proposed which divides the total population into six groups: susceptible individuals (S), free criminals (C), criminals arrested in jail  $(J_a)$ , convicted criminals  $(C_v)$ , judges (J) and police officers (P). It is assumed that susceptible individuals, honest police, and judges can become criminals through a social contagion process with criminals. It is also assumed that the population is well mixed and there is no difference in rates of transmission of crime based age behavior, occupation, criminal history. For instance, a young child or an individual with no deficit of resources or socially inactive is less prone to become a criminal than an individual with a criminal history or an individual with a deficit of resources. However, the rates are taken to be average for different populations. In addition, it is assumed that the total population N = S(t) + C(t) +  $C_v(t)$  + J(t) +  $J_a(t)$ + P(t) is constant as immigration and emigration from and to the other regions are neglected.

### Hypothesis of flow between sub-populations

The flow of individuals from one class to another is governed by the following principles:

• Susceptible individuals are non-criminals who can

transform into the criminal class through an interaction with the criminals, to the police compartment by becoming a police officer, and also to the compartment of judges.

- An individual in criminal sub-population can flow to the jail sub-population  $J_a(t)$  if an honest police officer arrests the criminal and takes him to jail.
- A jailed criminal can transform to the convicted sub-population  $C_v(t)$  if an honest judge in charge of the trial finds the individual guilty. An individual in the jail population  $J_a(t)$  becomes a susceptible individual at a flow rate  $r_1$  that depends implicitly on the corrupt judges and police officers.
- An individual in the convicted sub-population  $C_v(t)$  can become susceptible at rate  $r_2$  on being released from jail. It is very likely that a convicted individual transforming to the susceptible population again flows back to the criminal sub-population immediately after getting released from jail but it is assumed that this doesn't happen initially.
- A judge can become a criminal by becoming corrupt in two different ways. The first one is by effective social contact with a criminal and the second one is by an implicit social contact with an individual in jail which may come through the external agents related to the jailed individual.
- A police officer can become a criminal by effective social contact with a criminal.

### Mathematical model

The dynamics of these classes is given by the following system of six nonlinear ordinary differential equations:

$$\begin{aligned} \frac{dS}{dt} &= \mu N - \beta_1 SC - (a_1 + a_2)S + r_1 J_a + r_2 C_v - \mu S \\ \frac{dC}{dt} &= \beta_1 SC - \beta_2 CP + \beta_6 JC + \beta_5 J J_a + \beta_4 PC - \mu C \\ \frac{dC_v}{dt} &= \beta_3 J J_a - r_2 C_v - \mu C_v \\ \frac{dJ}{dt} &= a_1 S - \beta_5 J J_a - \beta_6 JC - \mu J \\ \frac{dJ_a}{dt} &= \beta_2 CP - \beta_3 J_a J - r_1 J_a - \mu J_a \\ \frac{dP}{dt} &= a_2 S - \beta_4 PC - \mu P \end{aligned}$$

Description of various parameters is as follows:

- $\mu$  is the birth/death rate.
- β<sub>1</sub> is the crime transmission rate from criminals to susceptible individuals which is related to the effectiveness of interaction of criminals and susceptibles in the spread of crime.

(2)

- $\beta_2$  is the rate at which criminals are caught by police and sent to jail which depends on the number of honest police officers available.
- $\beta_3$  is the rate at which jailed criminals get convicted by an honest judge.
- β<sub>4</sub> is the rate at which crime transmits from criminals to police officers.
- $\beta_5$  is the rate at which criminals corrupt police officers due to factors such as bribe, etc hence implicitly leading to the judges becoming corrupt and conversion of judges to criminals.
- $\beta_6$  is the rate at which criminals corrupt judges.
- $a_1$  and  $a_2$  are the growth rates for judges and police officers, respectively, which are dependent on the susceptible population. Factors affecting this rate include the availability of vacancies in the two professions and education awareness among the susceptible population.
- $r_1$  is the rate at which jailed criminals flow to the susceptible class. This is possible in two ways, one, when the police officer is corrupt, two, if the criminal was innocent and was jailed due to misunderstanding or mistake of police officers.
- $r_2$  is the rate at which convicted criminals flow to the susceptible class. This again is possible in two ways, one, when judges become corrupt, two, when the criminal was actually innocent.

It has to be noticed that the transition rates are assumed constant to study the evolution of criminal dynamics over a finite time.

Now, as it is assumed that the total population is constant at a given time t, i.e.,

$$N(t) = S(t) + C(t) + C_v(t) + J(t) + J_a(t) + P(t)$$
(3)

which implies that,

$$\frac{dN}{dt} = 0$$
, with initial population size,  $N(0) = N_0 > 0$ .

It can be proved that the model system (2) has unique and bounded solutions.

## Crime-free equilibrium and Reproductive number

We can calculate the equilibrium points by putting the right-hand side of all differential equations of the model system (2). We will get very long and complicated expressions. But here we are more interested in crime-free equilibrium which would help us in analyzing how society can be made completely free of crime. The crime-free equilibrium is given as:

$$CFE = (S^e, 0, 0, 0, J^e, P^e)$$
$$= \left(\frac{\mu N}{a_1 + a_2 + \mu}, 0, 0, 0, \frac{a_1 N}{a_1 + a_2 + \mu}, \frac{a_2 N}{a_1 + a_2 + \mu}\right)$$

In epidemiology, the basic reproduction number  $(R_0)$  is used to measure the transmission potential of a disease. It is the average number of secondary infections produced by a typical case of an infection in a population where everyone is susceptible.  $R_0$  helps in determining if an emerging infectious disease can spread in a population. In commonly used infection models, when  $R_0 > 1$  the infection will be able to start spreading in a population, but not if  $R_0 < 1$ . The number  $R_0$  has also been used in models of social epidemics where it would help in determining if the crime would spread for a given number of individuals in each class of sub-population.

The basic reproductive number  $R_0$  for the mathematical model (2) can be obtained by using the nextgeneration technique, where  $R_0$  is the eigenvalue of the next generation matrix such that all other eigenvalues of the next-generation matrix have a modulus less than  $R_0$ .

On applying the next generation technique we obtain  $R_0$  as,

$$R_0 = \frac{N}{D} \tag{5}$$

where, N=J<sup>e</sup>( $A_{13}\mu r_2 + 2A_{13}\mu^2 + \beta_1\mu B_{12} + r_1\mu^2\beta_1 + a_1A_{36}r_2 + 2a_1\mu A_{36} + a_1\beta_6B_{12} + a_1\mu\beta_6r_1 + A_{34}a_2r_2 + 2A_{34}a_2\mu + \beta_4a_2B_{12} + r_1\mu\beta_4a_2 + A_{25}a_2r_2 + \beta_5J^e\beta_2a_2\mu)$ and,

 $D = \beta_2 a_2 J^{e_2} \beta_3 r_2 + 2\beta_2 a_2 J^{e_2} \mu \beta_3 + \beta_2 a_2 J^e r_1 r_2 + \beta_2 a_2 J^e r_1 \mu + a_1 \mu \beta_3 J^e r_2 + 2a_1 \mu^2 \beta_3 J^e + a_1 \mu r_1 r_2 + a_1 \mu^2 r_1$ where,  $A_{ij} = \beta_i \beta_j J^e$  and  $B_{ij} = r_i r_j$ 

On doing stability analysis it is found that the crimefree equilibrium (CFE) is locally asymptotically stable for  $R_0 < 1$  and unstable for  $R_0 > 1$ . In epidemiology,  $R_0$ can be interpreted as the ratio of secondary cases from an infected individual to the number of recovering individuals. On drawing analogy, we get that when  $R_0 < 1$ the number of recovering individuals is more than the individuals becoming criminal, hence, the criminality eventually disappears. On the other hand, when  $R_0 > 1$ , the number of new criminal cases is more than the individuals becoming susceptible, hence, the criminality persists. It can be noticed that  $R_0$  can be decreased by decreasing N, which can be done by decreasing  $\beta_1$ ,  $\beta_4$ ,  $\beta_5$  and  $\beta_6$ , and increasing D, which can be done by increasing  $\beta_2$ and  $\beta_3$ . In words, the crime can disappear if the rate at which non-criminals (susceptibles, police, and judges) are changing to criminals is reduced and the rate at which criminals are jailed and convicted is increased.

### Concluding remarks

In this paper, the dynamics of the criminal and noncriminal population is modeled based on the assumption that crime is a social epidemic process. A very comprehensive model is presented by dividing the population into six classes considering the criminal class, common population and the law officials. A system of six nonlinear differential equations describes the transformation of individuals from one class to another. One thing to notice in this model is that the birth and death rate is considered to be equal which is not a real case scenario. It is also assumed that the population is well mixed and values of parameters don't depend on the characteristics of the population. But in reality, the degree of spread of crime may vary depending on the immunity of the susceptibles and contagiousness of the infected. This is resolved by taking the average values of the parameters. To analyze the condition for the extinction of criminality the threshold parameter  $R_0$  is computed using the next generation technique. From stability analysis, we get that society becomes crime-free when  $R_0 < 1$ . On the other hand, crime persists when  $R_0 > 1$ . The analysis of  $R_0$  shows that its value can be increased or decreased by varying the rates of transition. The parameter  $\beta_1$  is very important to make the society crime-free as it is related to the transition from the susceptible class to the criminal class. It can be decreased by avoiding contact between criminals and susceptibles which can be achieved by ensuring the proper availability of resources to the susceptibles so that they don't take criminal activities to make their living. The parameter  $\beta_4$  which is related to the transition of police officers to criminal class hits two targets with one arrow. One is, increase in criminals due to the transition of police officers to criminals, and the other is the increase in criminals due to an increase in interaction with the susceptibles because of the scarce availability of honest police officers to maintain law and order and to jail the criminals. Other parameters of equal importance as that of  $\beta_4$  are  $\beta_5$  and  $\beta_6$  as these are related to the transition of the judges to the criminal class. Intuitively, it can be inferred that crime can be reduced by increased recruitment of honest law officials. This is not directly related to the rates  $a_1$  and  $a_2$ , as they denote recruitment rate in general and do not specify if the law official is honest or corrupt. So  $a_1$  and  $a_2$  do not help directly in reducing the threshold parameter. This paper also discusses a very effective way of estimating the values of parameters using finite difference approximations for the derivatives and average values for the fluxes in the model which wouldn't be discussed here. It can be inferred that society can be made crime-free by improving the implementation of the law, reducing the interaction between criminals and susceptibles, and by improving the availability of resources to all the individuals.

# III. CRIME AND PUNISHMENT [?]

## Introduction

This paper takes a population based approach rooted in the models that have been mainly developed to model the spread of disease. Its main aim is to study the spread of crime and dynamics of incarceration and compute and analyse the thresholds involved in getting a crime-free society with an optimal rate of incarceration. The models allow us to investigate simultaneously the systematic effects of contagion, desistance, incarceration rates of first time and repeat offenders, prison term length, and criminal rehabilitation/redemption on long-run crime and incarceration outcomes.

### Assumptions

- A homogeneous population is assumed and the differences in ages are not taken into account.
- The crimes are not differentiated on the basis of their frequency or severity.
- The model does not take into consideration the variety of actions taken at different stages of the criminal justice system.
- Empirical data has not been included in the paper.

## **Population Structure**

The population has been divided into 5 compartments based on criminal activity at a given point of time:

- X Not Criminally Active
- $C_1$  Criminally Active but never incarcerated in past
- $\bullet~I$  Incarce rated
- R Once incarcerated but not Criminally active
- $C_2$  Once incarcerated and again criminally active

# Models and Analysis

The paper analyses six model systems with different levels of complexity. The simplest models (1 and 2) are basic three dimensional models that distinguish between criminally inactive (X), criminally active (C) and incarcerated (I). The model systems 3 and 4 are further improved five-dimensional models that capture prisoner reentry (return from jail). The further complex model systems 5 and 6 are nine-dimensional and introduce temporal distinctions between different spells of incarceration in order to analyse the effect of three-strike policy.

# Model 1: Simplified 3-dimensional Model

This three-dimensional model has only three compartments: the not criminally active (X), the criminally active (C) and the Incarcerated (I). the flow into criminal population is mainly due to the contagion parameter that is, due to contact between criminal and non-criminal population.

# Parameters for Models 1 and 2

 $\alpha_{11}$ : Contagion parameters of criminal behavior

 $\beta$ : Rate at which criminals discontinue criminal habits(desistance)

 $\gamma$ : Rate at which criminals are incarcerated

 $\epsilon$  : Rate at which incarce rated individuals are released and assimilate back into the society

 $\delta$  : Rate at which incarce rated are released and return to criminal life

System 1

$$\dot{X} = \beta C - \alpha_{11} \frac{XC}{(N-1)} + \epsilon I \tag{6}$$

$$\dot{C} = -\beta C + \alpha_{11} \frac{XC}{(N-1)} - \gamma C + \delta I \tag{7}$$

$$\dot{I} = \gamma C - I(\delta + \epsilon) \tag{8}$$

$$X + C + I = N \tag{9}$$

Using equation 9, the Model system 1 can be reduced to a two dimensional system 2 involving only the variables C and I.

System 2

$$\dot{C} = -\beta C + \alpha_{11} \frac{(N - C - I)C}{(N - 1)} - \gamma C + \delta I \qquad (10)$$

$$\dot{I} = \gamma C - I(\delta + \epsilon) \tag{11}$$

Analysis

The Systems 1 and 2 have been analysed using the phase planar representation of the two-dimensional system 2 as well as using a Lyapunov function to find the tipping point that distinguishes convergence to high-crime equilibrium from convergence to low-crime equilibrium.

Both the ways of analysis lead to the same results i.e. when  $(\beta + \gamma) - \alpha_{11} > 0$  and

$$\frac{(\beta + \gamma) - \alpha_{11}}{\delta} > \frac{\gamma}{\delta + \epsilon} \tag{12}$$

then, the system converges to a crime-free equilibrium, otherwise the system tends to a high crime endemic equilibrium. Hence, equation 12 defines the **tipping point** between high-crime and low-crime equilibrium.

# Interpretation

Equation 12 can be written as

$$\frac{\alpha_{11}}{\beta + \gamma(\frac{\epsilon}{\epsilon + \delta})} < 1 \tag{13}$$

In the above inequality, the numerator is the input rate to the criminal class. The denominator is the rate out of the criminal class with the incarceration rate  $\gamma$  multiplied by a non-recidivism factor  $\left(\frac{\epsilon}{\delta+\epsilon}\right)$  and  $\beta$ , the desistance rate. The impact of incarceration with respect to the rehabilitation rate is attenuated by the fact that some of those incarcerated will return to crime. Hence, the numerator represents the inflow into the criminal population while the denominator is the outflow. When inflow becomes less than the outflow, the society approaches a crime-free scenario.

This ratio is called the *basic reproduction number*  $R_0$  in demography and epidemiology which as defined before, can be interpreted as the number of people a criminally active person can seduce to criminal activity during his or her active period.

# Model 2 : Beyond Contagion, a More Complex 3-Dimensional Model

In this model, the assumption that people turn into crime only through contact with criminal class has been relaxed. A new parameter  $\alpha_{10}$  has been introduced that represents the rate of conversion from criminally inactive to active without any influence from criminal class.

System 3

$$\dot{X} = \beta C - \alpha_{11} \frac{XC}{(N-1)} - \alpha_{10} X + \epsilon I$$
(14)

$$\dot{C} = -\beta C + \alpha_{11} \frac{XC}{(N-1)} + \alpha_{10} X - \gamma C + \delta I \qquad (15)$$

$$\dot{I} = \gamma C - I(\delta + \epsilon) \tag{16}$$

$$X + C + I = N \tag{17}$$

The reduced System 3 now becomes

$$\dot{C} = -\beta C + \alpha_{11} \frac{(N - C - I)C}{(N - 1)} - \gamma C + \delta I + \alpha_{10}(N - C - I)$$
(18)

$$I = \gamma C - I(\delta + \epsilon) \tag{19}$$

# **Analysis and Interpretation**

The analysis of the system is done on similar lines with the System 1 and 2. The analysis shows that the origin is no longer a stable point in any case. Hence, there is no longer a crime-free equilibrium, since some proportion of law-abiding citizens will turn to crime on their own. It is shown that the previous endemic equilibrium bifurcates to a low-crime equilibrium and the previous endemic equilibrium bifurcates to a high-crime equilibrium.

The threshold condition for the stability of crime-free equilibrium for Model 1 still holds for the low-crime equilibrium in the bifurcated system,

$$\frac{(\beta + \gamma) - \alpha}{\delta} > \frac{\gamma}{\delta + \epsilon} \tag{20}$$

# (Here $\alpha = \alpha_{11} + \alpha_{10}$ )

Otherwise, high-crime equilibrium is globally stable.Therefore, when some people have a propensity to commit crimes independent of contagion effects, crime will always persist, but there are still two distinctive longrun equilibrium levels of crime.

# Model 3: Simplified Five Dimensional Model

The 3-dimensional models have been extended to a 5dimensional model by adding 2 more compartments and more complexities.

- Repeat offenders have been added to the model. Those who return to crime after incarceration are added to a different compartment  $(C_2)$  from those who have not been incarcerated previously  $(C_1)$ .
- A transient post-prison compartment R has been added that comprises of recently released people.

# Parameters for Models 3 and 4

 $\alpha_1$ : The rate at which individuals move from state X (not criminally active) to C (criminally active but not incarcerated).  $\alpha_1$  can be decomposed into  $\alpha_{11}$  that represents the contagion rate that is, flow into criminal activity on having contact with criminally active people and  $\alpha_{10}$  representing the rate at which criminally inactive people turn into crime without any contact with criminal class.

 $\beta_1$ : The rate at which people move from state  $C_1$ (criminally active but not incarcerated) to X(not criminally active). This represents the rate of desistance.

 $\gamma_1$ : The rate at which individuals move from state  $C_1({\rm criminally} \mbox{ active but not incarcerated})$  to  $I({\rm incarcerated}).$  This parameter represents the rate of incarceration.

r: The rate at which individuals move from state I(incarcerated) to R(formerly incarcerated but not criminally active). This parameter represents the rate at which criminals are released from prison/jail.

 $\alpha_2$ : The rate at which individuals move from state R(formerly incarcerated but not criminally-active) to  $C_2$ (formerly incarcerated and criminally active). It is a rate at which people resume criminally active and captures recidivism.

 $\gamma_2$ : The rate at which formerly incarcerated criminals $(C_2)$  are incarcerated again.

 $\beta_2$ : The rate at which formerly incarcerated criminals $(C_2)$  move to formerly incarcerated but not criminally active(R). This parameter captures the rate of desistance or effect of prison i.e specific determence.

 $\epsilon$ : The rate at which individuals move from R (formerly incarcerated but not criminally active) to X(not criminally active). It is similar to  $\beta_2$  except the state it leads to.

# System 4

$$\dot{X} = \beta_1 C_1 - \alpha_{11} \frac{X(C_1 + C_2)}{(N-1)} - \alpha_{10} X + \epsilon R \qquad (21)$$

$$\dot{C}_1 = -\beta_1 C_1 + \alpha_{11} \frac{X(C_1 + C_2)}{(N-1)} + \alpha_{10} X - \gamma_1 C_1 \quad (22)$$

$$\dot{I} = \gamma_1 C_1 + \gamma_2 C_2 - rI \tag{23}$$

$$\dot{R} = rI + \beta_2 C_2 - \alpha_{21} \frac{X(C_1 + C_2)}{(N-1)} - \alpha_{20} R - \epsilon R \quad (24)$$

$$\dot{C}_2 = \alpha_{21} \frac{X(C_1 + C_2)}{(N-1)} + \alpha_{20}R - \gamma_2 C_2 - \beta_2 C_2 \qquad (25)$$

$$X + C_1 + C_2 + I + R = N (26)$$

# Analysis

The simple 5-D Model assumes that  $\alpha_{10} = \alpha_{21} = 0$ . With this assumption, it has been shown that the basic reproduction rate comes out to be

$$R_0 = \frac{\alpha_{11}}{\beta_1 + \gamma_1} \left( 1 + \frac{\alpha_{20}\gamma_1}{(\gamma_2 + \beta_2)\epsilon} \right)$$

This ratio has been calculated using Lyapunov functions and it defines the threshold condition for the system. When this ratio is less than 1, the society approaches a crime-free equilibrium similar to Model 1.

# Interpretation

The basic reproduction ratio  $R_0$  can be interpreted as the input/output ratio of movement from X to C. The numerator in both the terms represents the inflow into criminal population while the denominator represents the different rates of outflow from from criminal population. Hence, when this ratio becomes less than 1, that is, the outflow becomes more than inflow, the society tends to a crime free scenario.

# Model 4: Beyond Contagion, More Complex 5-Dimensional Model

This model is similar to Model 3 and includes the possibility that people can turn into crime on their own. The System remains same as system 4 but here  $\alpha_{10}$  and  $\alpha_{21}$  are not equal to zero. As in the case of 3-Dimensional Model, the system here too bifurcates to a low-crime and highcrime equilibrium and the origin is no longer a stable point. However, the tipping point remains the same.

# Policy-Related Implications of 5-D Model

**Incarceration/Law Enforcement**: Expression for  $R_0$  and the threshold condition tell us that in order to have a crime free society, the contagion/transmission factors given by  $\alpha$ 's in the numerator need to be small and the retreat from crime rates given by  $\beta$  and  $\epsilon$  should

be large. In addition,  $\gamma_1$  appears in the numerator suggesting if first time offenders are incarcerated too much, result can be counter-productive and  $\gamma_2$  term in the denominator suggests that incarcerating the repeat offenders more is desirable. To come to some conclusive values of  $\gamma_1$  and  $\gamma_2$  it has been shown that, when recidivism is smaller than rehabilitation and redemption, increasing  $\gamma_1$  does reduce crime in long run. However, if recidivism is greater than rehabilitation, increasing  $\gamma_1$  leads to high crime.

**Prison Term Length:** The prison term length is given by D = 1/r where r is the rate of prison release. The threshold condition does not contain any term of r, therefore, the prison term length has no direct effect on reduction of crime in society but it does appear in the expression for equilibrium populations. In the long run, a low value of r(longer prison terms) lead to a greater prison population reducing the number of criminally active individuals at a time.

**Long-Term sentences**: The effect of slowing the prison release is analysed by taking the limiting case where r is taken to zero i.e. no one is ever released from prison. In this case, the compartments  $C_2$  and R disappear and the 5-Dimensional Model reduces to a 3-Dimensional model whose crime free threshold is  $R_0 = \frac{\alpha_{11}}{\beta_1 + \gamma_1}$ . When  $R_0 > 1$ , the only solution obtained is N = I, hence the entire population is incarcerated eventually. On the other hand, when  $R_0 < 1$ , the system reaches a crime free equilibrium where, the criminal population is either incarcerated or desists.

**Desistance:** To quantify the effect of desistance, partial derivatives of  $R_0$  with respect to desistance parameters  $\beta_1$  and  $\beta_2$  are computed which turn out to be negative, and therefore it can be concluded that social interventions targeting desistance of active criminals unambiguously decrease crime.

# CONCLUSION

Mathematical modeling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application. This review provides an example of how we can intertwine mathematics into sociology and seek solutions to social problems.

The two-compartmental model investigates the effect of law enforcement in the society and gives us value of law enforcement parameter crucial for controlling crime. The second paper investigates many other models to observe the effect of punishment on the spread of crime. It projects light on some unexpected results of the model such as the prison term length has no effect in reduction of crime. It also suggests that it is important to distinguish between first time and repeat offenders and that incarcerating repeat offenders is more effective.

Social experiments can be conducted to verify these mathematical results which can prove to be very useful in drafting policies.

# SCENE 3:



# LEONARD SUSSKIND'S WORST NIGHTMARE

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# PART 1: Analysis of Vela Pulsar

In the first part of the project, the Vela Pulsar (one of the brightest pulsar {at radio frequencies} in the sky with a period of 89.33 milliseconds – the only pulsar visible in both optical and X – ray range) has been analysed using the voltage data from the Ooty Radio Telescope (ORT) — North and South apertures. The project mainly consists of three parts: Signal statistics, Time and Frequency domain Analysis of signal and finally discovery or detection of the presence of pulsar from given data along with calculation of some of its relevant parameters. This data is just a sample of the voltage signal obtained from a 1 second observation of the Vela pulsar (PSR B0833-45) with the North and South antenna of the ORT, which has been conducted at 326.5 MHz with a bandwidth of 16.5 MHz.

# SIGNAL STATISTICS:

Here, the voltage signal characteristics (voltage as function of time) have been plotted and observed to have Gaussian Distribution as expected. Then, mean and standard deviation of the system have been calculated for both the northern and southern half voltage values as:

 $\mu_{North}$  = 3.482,  $\sigma_{North}$  = 28.016

 $\mu_{South}$  = 0.737,  $\sigma_{South}$  = 29.832

X-ray Image of the Vela Pulsar as captured by the Chandra X-ray Telescope.

The large mean in the signal for the North antenna has been observed, implying the presence of instrumental error of the antenna receiver system due to the biasing of circuits that introduces this DC offset.

Further, the power signal characteristics have been plotted and observed to have exponentially decaying distribution as expected.

# TIME - FREQUENCY DOMAIN ANALYSIS OF SIGNAL:

In this section: At first, signal is analysed in the frequency domain to see the power spectrum of the voltage signal to identify any signs of aliasing in the frequency domain. Fast Fourier Transform (FFT) has been used to obtain the average power spectrum of the voltage signal. It turns out that DC channel power is comparatively larger in the northern signal, consistent with the fact that the norther signal voltage mean is relatively larger. However, once the DC offset voltage is removed, the power spectrum smoothly tapers off to zero at both the edges of the band, indicating that aliasing is very minimal for these receivers.



Average Power Spectral Density Curve after removing the DC Offset of receiver.

Secondly, the first signs of the pulsar signal are detected by looking at the dynamic spectrum of the signal, i.e., the power in the signal as a function of time and frequency. In this spectrum, a pulsed signal appears earliest at the high frequencies, and gradually appears later at lower frequencies. This frequency dependent delay is a characteristic sign of a signal dispersed in the **Interstellar Medium (ISM).** The observed delays are then used to estimate the Dispersion Measure (DM) along the line of sight to the pulsar.

# From the crude analysis of the dynamic spectrum, DM turns out around 64.31 ${ m cc}^{-1}$

# **DYNAMIC SPECTRUM:**

The first signs of the pulsar signal are detected by looking at the dynamic spectrum of signal, i.e., the power in the signal as a function of time and frequency. In the images shown below, y-axis is frequency in MHz, x-axis is time in milliseconds and the colour code indicates the signal power for a particular frequency and time sample. A pulsed signal appears earliest at the high frequencies, and gradually appears later at lower frequencies. This frequency dependent delay is a characteristic sign

of a signal dispersed in the interstellar medium. The observed delays can be used to estimate the Dispersion Measure (DM) along the line of sight to the pulsar.



Dynamic spectra obtained for combined and consecutive data set.

# **DISCOVERY (DETECTION) OF PULSARS:**

Assuming that the electron density number is constant and using the fact that the ISM disperses photons propagating through it as a function of photon frequency ( $\nu$ ) similar to a prism, the DM value is used to calculate **the distance of the pulsar from Earth (d)** which comes out as -

d =2.143kpc, taking  $n_e = 0.03cc^{-1}$ 

The signal is now de-dispersed to correct for the frequency dependent term by applying the required time delays to the frequency channels. This shifting of time delay results in clearly visible single pulses above the background noise with significantly high Signal to Noise Ratio (SNR).

Ultimately, in order to find the rotation period of the pulsar, the arrival times of available high SNR individual pulses are found by fitting them with Gaussian Curves. Based on the period estimated, the entire time series is folded with the pulsar period to obtain an average profile for the pulsar as:



The Time Folding has been done at Pulsar period of 92ms. The plot clearly shows the presence of a pulsar.

In closing, the numerical results obtained about the pulsar are stated as:

- $DM \approx 64.31 pc/cc$  as obtained from the Dynamic Spectrum.
- Pulsar Distance  $\approx$  2.143 kpc, using constant Electron Density Number  $n_e = 0.03cc^{-1}$ . But the actual distance of the pulsar is 294 pc. This error has occurred because of the presence of Gum Nebula.
- Pulsar period obtained is 93ms while the actual period is 83.33ms.

Last but not the least, crude analysis is supposed to cause the erroneous value of DM and hence requirement of sophistication in the current method of analysis has been suggested to the prospective analysists.

# Part 2: Sea surface Interferometer

This experiment aims at studying the interference pattern formed by Sea Interferometer using simple domestic setup which includes smartphone, laptop with appropriate software installed to get the readings, wooden stick, measuring tape etc.



The setup is made as shown above and height of the smartphone is adjusted accordingly so position could be marked on the antenna to get the corresponding RSSI value at that point. This is one of the oldest methods used in Radio Astronomy. It uses the reflection of radio waves from sea surface and obtaining the interference pattern caused by the direct and the reflected wavefront from the Celestial source. The receiver was placed on a cliff with the sea facing in front. The Earth's sidereal rotation causes the source to move around in sky, and thus showing variation in the interference pattern observed at the receiver. Below, is a domestic set-up of the experiment



We are utilising the Wi-Fi feature in both the devices, as Wi-Fi signals typically operate on  $v_0 = 2.4$ GHz (in this case) with a bandwidth of  $\Delta v = 20$ MHz. The fractional bandwidth is much less than unity  $\frac{\Delta v}{v} \ll 1$ , so we can consider it to be a monochromatic radio frequency (RF) wave, having a wavelength of  $\lambda_0 = \frac{c}{v_0} = 12.5 \ cm$ . The phase difference between  $R_1$  and  $R_2$  is:

$$\Delta \varphi = \frac{2\pi}{\lambda} \left( \frac{2h}{(D^2 + h^2)^{\frac{3}{2}}} \right) y + \pi$$

Net electric field at R is:

 $E_{net} = E_1 + E_2 = E_0 cos\omega t + E_0 (cos\omega t + \Delta\varphi) = 2E_0 \cos\left(\frac{2\omega t + \Delta\varphi}{2}\right) \cos\left(\frac{\Delta\varphi}{2}\right)$ 

So, net intensity recorded at R is:

$$I_{net} = \frac{I_0}{2} (1 + \cos\Delta\varphi) = \frac{I_0}{2} (1 - \cos\left(\frac{2h}{(D^2 + h^2)^{\frac{3}{2}}}\right))$$

It's a sinusoidal interference pattern.

The steady RSSI power recorded at receiver was noted down for each value of y, and was plotted using Python, which is shown below:



From this fitting we can see the interference is periodic with about length period 24 cm, it is centred at -38 dBm and has a 11 dBm peak-to-peak variation.

# **Results:**

The interference pattern is observed to follow sine pattern as expected. The wavelength of the radio wave determined from the graph above is  $\Delta y = 24$ cm. Putting the values D = 1 m and h = 0.28 m,

$$\lambda = \frac{(2h)}{(D^2 + h^2)^{\frac{3}{2}}} \Delta y = \frac{2 \times 0.28}{(1^2 + 0.28^2)^{\frac{3}{2}}} \times 24 = 12cm$$

Which closely matches with actual wavelength of 12.5cm for standard Wi-Fi signal.

Some key points observed were:

- 1. Near field effects were neglected as aperture of antenna in smartphone is very small.
- 2. RSSI value seems fluctuating at each point but gets stable after sometime which is then recorded.
- 3. The observed graph nearly follows cosine interpolation but deviations get large at greater heights. This is because the reflected wave is travelling a much greater distance than direct wave, and due to the inverse square law; the reflected wave arrives with comparatively lower intensity that affects the interference pattern.
- 4. This was a crude setup easily influenced by environmental variables thus leaving room for large errors.

This method could be very useful in finding wavelength, frequency and position of any celestial radio source and thus was used to discover some bright radio objects in sky in the past.

Summarised by:

Aishwarya David & Ashutosh Mishra (2<sup>nd</sup> Physics)

# SCENE 4:



# THE HAMLETIAN 1:30 PROBLEM

DOODLE CREDITS: GARGI SINGH DOODLE CONCEPT: SWAPNILA CHAKRABARTY

# Historical Ideas on absolute and relative motions

Note: This article aims to deal with some major historical arguments so as to appreciate them and the fundamental questions they concern. Relatively modern developments in physics have offered us new and better conceptual understanding and theoretical frameworks and hence rendered most of them untrue in general. Yet, understanding them offers insights also simultaneously helping to develop the necessary background needed to understand the modern theories.

Absolute and relative in the context of motion concerns dealing with frames of reference used to describe the motion, the relationship that exists between observer and the observed and even the concept of empty space itself among others. One thing which we all can immediately observe is the relative property of linear motion. It is evident that in different inertial frames we measure different values for the velocity of a particular train. But now if we are asked to measure how fast or slow a 2nd train is moving with respect to the first, we all agree to one value. In-fact the distances between bodies at any instant of time will always be measured the same in any inertial frame. From this fact, the former one follows. These statements are a natural result of geometric, mathematical arguments and the definition of the quantities concerned. In the modern context, the vector picture gives a simple and quickly understandable proof.

Thus, we see, it is not sensible to ask what the velocity is. 'What is the relative velocity of an entity with respect to another or to a particular reference frame?' is the more sensible question. Because that will have a clear, unique frame-independent value. This was understood even by early philosophers in the Newtonian times and even before. Yet most of them believed that states of true motion and rest exist and they are different from these relative motions that seem to add to the question of motion - "with respect to what".

As to why the above arguments failed to convince philosophers that states of true motion and rest don't exist, we'll see now. Above, we carefully ignored talking about acceleration - the change in velocity and/or direction of motion. Before and around Newton's era, it was quite unclear as to what causes this acceleration required to make the planets move in great circles. Does it even require a 'cause' or is it the natural tendency of planets? Some earthly observations like whirling a stone around with a rope helps. We do see that this requires force and a bigger stone requires greater force to move it around at the same speed. And when we fling it, the stone leaves away tangentially. This might be true at astronomical scales also. So, slowly but firmly it was established that some force is indeed required for the planets to move around.

Copernican heliocentric theory proved that the predictive power is not lost by taking some other point (the sun) to be fixed for describing trajectories of planets. Hence, for giving preference to a particular reference frame (the Earth), successful astronomical predictions are not sufficient. There should be some other, maybe more fundamental criteria(s). These were thought to be the **true physical causes of motion**. Philosophers agreed that the heliocentric picture was uniquely suited to describe planetary motions because the Sun has an important role to play as the true cause of motion of the planets. Newton thought that a coherent account of force and motion requires a background space, with places reaching to infinity and maintaining given positions with respect to each other. He gave the name **absolute space** to space understood in this sense, as the universal frame of reference with respect to which the displacements of bodies constitute their 'true' motions. He argued that it can be known by

its properties, causes and effects.

However, the difficulty imposed upon this view stems from Newton's own conception of force. It is taken to be the 'power' to accelerate bodies and hence measured in terms of those accelerations that it is thought to produce.

If it is determined that all the accelerations within a system of interacting bodies are only proportional to the forces impressed by the bodies within the system on each other, it means that all bodies in the system experience no external force. In this case, the common centre of mass is found to be non-accelerating. And this is in agreement from what can be derived from Newton's laws that the centre of mass doesn't accelerate as seen from an inertial frame when no external force is present. Now, since different frames of reference measure accelerations to be the same and hence forces too, these true causes of motions furnish us no way to determine the *true* velocities or *true* state of rest of the particles or the centre of mass of the system. Since each frame will give a different value for the velocities and no one can be considered special or true as they all are measuring the forces to be the same.

The idea of the luminiferous aether is also related to the notion of absolute space. It was thought that object's and indeed light's true speed is with respect to this aether. But aether was more needed to explain the propagation of light waves in vacuum rather than being the concrete form in which "absolute space" exists. Only by the turn of the 19th century with the Michelson-Morley experiment, the existence of both aether and an absolute space began to be doubted strongly by scientists.

In conclusion, we note how the existence of infinite equivalent inertial frames, that follows from Newton's laws, can be thought of as a very strong argument against the existence of a background absolute space in reference to which bodies move in a true sense.

But still the issue is not completely resolved. The arguments around the absoluteness or relativeness of **Rotational motion** are more involved. The distinguishing features here are the non-inertial frames and their properties. First let us think on a few points about rotational motion.

An object is rotating about an axis with a particular speed of rotation in empty space. Does it need a force to continue this indefinitely? Let us also assume that the axis is fixed with respect to an inertial frame. Now? The answer is that it still depends on the position of the axis.

- If it passes through the centre of mass of the body then it doesn't need an external force (think about astronauts when they hit something-a pen maybe, inside a shuttle and it starts rotating)
- Otherwise, it does need a force, since the centre of mass will need to move around the axis and hence undergoes acceleration. And we know this cannot happen by Newton's laws, without an external force.

What about the speed of rotation? This characterises the rotation just like velocity in pure rectilinear motion. We simply say how much the angle of a rotating object or a point within it changes in a given time, as measured from the axis and that can give us its speed of rotation. In physical situations, measuring how much time it takes for a full 360-degree rotation we find the period of rotation. Earth rotates about its axis: 24-hour cycle. How do we measure this? Taking the Sun as a reference we can measure the time between two days when the Sun is exactly overhead. Done. A **solar day.** But something's wrong in this way. Carefully see the diagram below:



Because Earth is also moving around the Sun, the next day the sun will be directly overhead at a little later time. That is, the Earth will have to rotate a little bit more (0.986°) than 360° (think about how to know when 360° happens). The method will give us a slower rotation speed of Earth and a bigger 'day'. In the same diagram we can see that with respect to a distant star, the measurement is better. Why is this so? Because these distant stars are very distant. The more distant means lesser effect of relative motion on angles. In this way we measure the **sidereal day**: nearly 3 minutes shorter than a solar day.

But this raises a more important question. Since rotation periods can be different, does this imply that just by moving to a different frame, the rotation can vanish? This would seem to imply that existence of rotation is not independent of everything else in space and rotation is not an inherent property of the body. Rather it is a relation to other nearby or distant bodies. We again move back to looking for causes and effects of rotation. Maybe they will give us new things to think about. The cause is clear - a force pulling towards the centre. This became clear in Newtonian time. The effects too, if you ever swinged round and round in a park swing, the force you feel that pushes you out. And indeed, if a light object is placed it gets thrown out. This endeavour to recede from the axis seems to exist in all rotations.

"The effects which distinguish absolute from relative motion are the forces of receding from the axis of circular motion. For there are no such forces in circular motion purely relative, but in a true and absolute circular motion, they are greater or less, according to the quantity of the motion." - Scholium, Book 1, Philosophiæ Naturalis Principia Mathematica

In light of this, now we try to understand two simple yet powerful arguments given by Newton. The aim of these arguments is often misinterpreted, as it was partially done by Ernst Mach. But for our present purposes, it suffices not to go in those details.

**The Bucket:** A Bucket filled with water is tied to a cord or rope and twirled many times. Then it is released from the grip. It is observed that as the bucket starts to spin, the water inside remains stationary. The endeavour to recede away from the axis, towards the walls of the bucket is not seen despite the fact that the relative motion between the walls and water is the greatest at this time. Slowly the motion of the bucket communicates itself to the water and it begins to recede and take a concave shape. After a while, water climbs to the maximum height on the walls of the bucket and there is no relative motion between the walls and water. Both are rotating together. From this it can be concluded that motion relative to the immediate surroundings has no mechanically significant value since water remains flat initially. Only slowly we begin to see the water receding, curving more and more, climbing to greater heights. This is the evidence that some mechanically significant rotation of the water is happening, and its measure is unique (the maximum height attained). Possibly it is with respect to a unique privileged frame (absolute space?).

**Two tied spheres:** Two globes at a given distance, tied with a cord are revolved about the common centre of mass of this system. From the tension in the cord, we can measure the endeavour of the globes to recede which is directly dependent on the speed of rotation. After that, if forces equal and opposite in direction are put on the globes, either they could increase the rotation speed, increasing the tension or decrease the speed and tension. And hence we can determine in which direction the forces ought to be applied to diminish the motion completely, thereby we can know the direction in which motion was taking place. Clockwise or counter-clockwise. All this can be done in an immense vacuum. With nothing other than the globes.

Now, if suppose the fixed stars are also present. Then the question is whether the motion is in our globes or in the fixed stars. Again, by measuring the tension we compare it with the former case, and

hence check whether it is what is required for that amount of rotation. If yes, then without doubt, the motion belongs to the globes.

Newton's arguments seem to argue in favour that an absolute space exists with respect to which he then suggests the existence of true motions. Figures like Ernst Mach and Descartes among others held opposite views and worked on them. Leibniz was absolutist about motion, considering true motion as the possession of 'active force'.

Mach critiques: The former Newton's experiment simply informs us that the relative rotation of the water with respect to the sides of the vessel produces no noticeable centrifugal forces, but they are produced by its relative rotations with respect to the mass of the Earth and other celestial bodies. A purely relational theory is still permissible. For the second experiment he seems to say: nobody is competent to say what would happen, or what would be possible, in a universe devoid of matter other than two globes.

A new kind of mechanics was sought in which effects attributed to inertia in the standard Newtonian mechanics: like the receding push on the water, would be results of motion and acceleration relative to massive bodies like fixed stars. The water in the bucket recedes because it is rotating relative to the fixed stars. Under a new physical law, this relative rotation would be the cause of the push. Einstein also interpreted Mach's principle as - inertia only originates in a kind of interaction between bodies. In a space devoid of all but one body, there would be no meaning of inertia for that body.

The theory of special relativity does away with the intuitive, implicit assumption that early philosophers made that time is absolute between two moving reference frames. So, the idea that velocities combine algebraically does not hold anymore. As with many more. Although the noticeable effects only arise at speeds comparable to the speed of light, yet our notions of space and time changed completely, more so with the general theory of relativity. So, the debate and attempts have taken new forms, dealing with new questions yet the fundamental issues concerning the true nature of motion and space remain to be resolved completely.

Deepanshu Bisht 2nd Physics

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# SCENE 5:



# LET US ASSUME THAT THE COW IS 'SPHERICAL' AND OF UNIFORM DENSITY.....

DOODLE CREDITS: GARGI SINGH DOODLE CONCEPT: SHALIKA YEKKAR

# Design of Quantum Circuits to Play Archery in a Quantum Computer

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Archery happens to be a very ancient game which originated primarily for hunting practices. It gradually became one of the most famous ways of recreation. Classical computers gave humans the opportunity to enjoy the digital version of the game. With the arrival of quantum computers, development of quantum games emerged to be a very intriguing and intellectually charging area of research. Here, we aim to develop a game of archery that can be played on a quantum computer, as quantum computers help achieve results much faster than classical ones and also intensify the complexities of the problem. We start with a very simple model of the computer playing the game by itself and then we gradually add more complexities to create a user-interactive and moving target model. We design simple quantum circuits which can be extrapolated to create efficient quantum archery engines. We verify the logic behind our quantum circuits on IBM quantum experience platform and give detailed description of our mathematical algorithm. Our final aim with this project is to put forward an idea efficient enough for further development of the game and subsequent addition of interesting complexities that will enhance the user's experience.

Keywords: Quantum Games, Quantum Archery, Quantum Circuit, IBM Quantum Experience

# I. INTRODUCTION

The multidisciplinary field of quantum computing strives to exploit some of the uncanny aspects of quantum mechanics to expand our computational horizons [1]. Research fraternity of quantum computation and information has specifically showcased its immense potential to create and develop games that can be played on currently available quantum computers [2]. Efficient application of the quantum principles of superposition and entanglement, researchers have come up with simple circuits to play games like shooting [3], chess [4], pong [4], sudoku [5] and bingo [6]. The Monty Hall problem [6] and Diner's Dilemma game [7] were successfully demonstrated with quantum circuits. A group of researchers also proposed a generalization of quantum prisoner's dilemma in the form of a quantum game [8]. Narula *et al.* gave an overview of designing quantum circuits in IBM's quantum experience platform [9]. Mahanti et al. suggested an interesting ideas of quantum robots playing games [10]. Eisert *et al.* suggested that developing games in quantum domain can be an interesting area to work in [11]

Game theory having various applications in scientific fields, we try to explore its quantum domain. Quantum game theory is an interdisciplinary field. It combines two completely separate disciplines greatly developed by von Neumann: game theory and quantum information theory. Like in classical game, in quantum game one can point out all basic notions that are used to define a classical game like a set of players, sets of strategies for the players and a payoff function. On the other hand, all these notions are expressed with the use of mathematical methods for quantum information like linear operators and unit vectors of a complex Hilbert space in accordance with the postulates of quantum mechanics. This new approach to the description of a game constitutes its strict generalization. For example, quantum game theory allows one to write any finite strategic game in the language of linear algebra in such a way that a classical game and its quantum counterpart are indistinguishable from game theory point of view. However, the most interesting property is that quantum information methods can give new, impossible in the classical case, scenarios of the game [12].

Game developing involves equal inputs of rigorous mathematical logic and good computational skills. In our project, we gradually add the complexities so that the project does not fizzle out, overwhelmed by the complexity of itself [13]. Greatly motivated by the works mentioned above, we propose a fundamental understanding of how we can play archery on a quantum computer. Our game follows the traditional concept of archery, of shooter trying to hit the target as precisely as possible (i.e., closer to the centre of target circle) [14]. Our focus remains on varying the final probabilities of hitting different points on the target. As we upgrade our levels and provide special advantages to the user, the game gets more and more interesting. If the algorithms suggested are worked upon, more appealing versions of the game can be developed by properly coding the algorithms

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which may finally yield a self-continuing game with the hardness increasing at each level.

# **II. THE GAME OF ARCHERY**

In the quantum model of our game of archery, we use a 5-pointer and a 7-pointer target instead of a classical 10-pointer one. We design 5 models of the game with different complexities, which are as follows:

- The first model is where the computer plays the game by itself using a 5-pointer target (corresponding to ideal physical conditions). The results here hint at the probabilities of hitting different circles of the target in perfect conditions.
- The second model has the computer playing the game by itself but in correspondence to varying physical conditions such that the final probabilities of hitting different circles in the 5-pointer target in the result vary in accordance to the instantaneously generated physical conditions.
- The third model is an user-interactive model where the user can choose a special position for his bow and arrow after being informed about the physical conditions and his target score.
- The fourth model is also an user-interactive model where the user is given the choice of a special bow and arrow. This special choice increases the user's probability of hitting a higher score (i.e., a circle closer to the centre) irrespective of the physical conditions but the initial score of the user is not zero in this case but negative (which increases as levels get harder). If the special bow and arrow option is not chosen, this level reduces to the level mentioned above where the user only chooses his bow and arrow position.
- The fifth model is where the computer plays the game by itself but the target is moving, therefore the probabilities of hitting different circles are constantly shifting in an explainable pattern. We use a 7-pointer target in this case.

# III. MATHEMATICAL MODELLING AND CORRESPONDING CIRCUITS

We start our modelling with the 5-pointer target. To, keep things simple, we assign the states of  $|100\rangle$ ,  $|010\rangle$ ,  $|110\rangle$ ,  $|001\rangle$ , and  $|101\rangle$  to the score of 1,2,3,4, and 5 respectively because when measured in classical bits,

- $|100\rangle$  corresponds to 1.
- $|010\rangle$  corresponds to 2.
- $|110\rangle$  corresponds to 3.

![](_page_59_Figure_13.jpeg)

FIG. 1: **Target in classical bits**. 101 corresponds to a score of 5, 100 corresponds to a score of 4, 011 corresponds to a score of 3, 010 corresponds to a score of 2 and 001 corresponds to a score of 1.

![](_page_59_Figure_15.jpeg)

FIG. 2: Circuit to eliminate unnecessary states: The first part eliminates  $|111\rangle$  state, the second part eliminates  $|011\rangle$  state and the third part eliminates  $|000\rangle$  state.

![](_page_59_Figure_17.jpeg)

FIG. 3: Circuit for III A: This circuit initially creates 8 superposed states with equal probabilities, then removes the extra states and returns a result that satisfies our logic.

- $|001\rangle$  corresponds to 4.
- $|101\rangle$  corresponds to 5.

A diagram of the target has been provided for reference (Fig. 1).

So, evidently we will be working on a 3-qubit system. However, a three-qubit system has 8 possible states, so we have to eliminate the states  $|000\rangle$ ,  $|011\rangle$  and  $|111\rangle$ . We use the Hadamard gate in the third qubit to put it in an initial state of  $|+\rangle$ , then we design the rest of the

![](_page_60_Figure_0.jpeg)

FIG. 4: Quantum circuit corresponding to III A. The extra q[3] used here is due to the fact that we have 3 control Hadamard gates and the gate provided by IBM QE only has a single qubit control at most. The first part after the 3 Hadamards eliminates the  $|111\rangle$  state. The second part eliminates the  $|011\rangle$  state and the third part eliminates the  $|000\rangle$  state. Anti-control qubit is made by preceding and succeeding the control qubit by an X gate.

| Value of | Input | Prob. of |
|----------|-------|----------|----------|----------|----------|----------|
| $\theta$ |       | getting  | getting  | getting  | getting  | getting  |
|          |       | 1        | 2        | 3        | 4        | 5        |
| 0        | 1     |          |          |          | 100      |          |
|          | 2     | 50       |          |          |          | 50       |
|          | 3     |          | 100      |          |          |          |
|          | 4     |          |          | 100      |          |          |
| $\pi/6$  | 1     | 3.125    | 6.25     | 0.44     | 87.05127 | 3.125    |
|          | 2     | 43.525   | 0.44     | 6.25     | 6.25     | 43.525   |
|          | 3     | 0.224    | 87.051   | 6.25     | 6.25     | 0.224    |
|          | 4     | 3.125    | 6.25     | 87       | 0.44     | 3.125    |
| $\pi/4$  | 1     | 6.25     | 12.5     | 2.144    | 72.855   | 6.25     |
| ,        | 2     | 36.42    | 2.144    | 12.5     | 12.5     | 1.07     |
|          | 3     | 1.07     | 72       | 12.5     | 12.5     | 1.07     |
|          | 4     | 6.25     | 12.5     | 72       | 2.14     | 6.25     |
| $\pi/3$  | 1     | 9.375    | 18.75    | 6.25     | 56.25    | 9.375    |
| ,        | 2     | 28.125   | 6.25     | 18.75    | 18.75    | 28.175   |
|          | 3     | 3.125    | 56.25    | 18.75    | 18.75    | 3.125    |
|          | 4     | 9.375    | 18.75    | 56.25    | 6.25     | 9.375    |
| $\pi/2$  | 1     | 12.5     | 25       | 25       | 25       | 12.5     |
| '        | 2     | 12.5     | 25       | 25       | 25       | 12.5     |
|          | 3     | 12.5     | 25       | 25       | 25       | 12.5     |
|          | 4     | 12.5     | 25       | 25       | 25       | 12.5     |
| $2\pi/3$ | 1     | 9.375    | 18.75    | 56.25    | 6.25     | 9.375    |
| ,        | 2     | 3.125    | 56.25    | 18.75    | 18.75    | 3.125    |
|          | 3     | 28.125   | 6.25     | 18.75    | 18.75    | 28.125   |
|          | 4     | 9.375    | 18.75    | 6.25     | 56.25    | 9.375    |
| $3\pi/4$ | 1     | 6.25     | 12.5     | 72.855   | 2.144    | 6.25     |
| ,        | 2     | 1.07     | 72.855   | 12.5     | 12.5     | 1.07233  |
|          | 3     | 36.4276  | 2.14466  | 12.5     | 12.5     | 36.4275  |
|          | 4     | 6.25     | 12.5     | 2.14466  | 72.855   | 6.25     |
| $5\pi/6$ | 1     | 3.125    | 6.25     | 87.05127 | 0.445    | 3.125    |
| ,        | 2     | 0.224    | 87.05    | 6.25     | 6.25     | 0.224    |
|          | 3     | 43.52    | 0.44     | 6.25     | 6.25     | 43.525   |
|          | 4     | 3.125    | 6.25     | 0.44     | 87.05    | 3.125    |
| $\pi$    | 1     |          |          | 100      |          |          |
|          | 2     |          | 100      |          |          |          |
|          | 3     | 50       |          |          |          | 50       |
|          | 4     |          |          |          | 100      |          |

TABLE I: Table for Probabilities of Scores for Different Inputs at Different Values of  $\theta$ . Here, 1 refers to  $|00\rangle$  input, 2 refers to  $|10\rangle$  input, 3 refers to  $|01\rangle$  input and 4 refers to  $|11\rangle$  input.

![](_page_60_Figure_4.jpeg)

FIG. 5: Final Probabilities of III A. The highest probability of middle scores and lesser probability for extreme scores.

![](_page_60_Figure_6.jpeg)

FIG. 6: Circuit for III B: In this circuit, we add the  $R_y$  gates to implement physical conditions computationally. The rest of the circuit does the same function of initializing 8 states with equal probability and eliminating the unnecessary ones.

circuit to eliminate the unnecessary states (Fig. 2). Part (1) has a control-control Hadamard (CCH) gate which eliminates the  $|111\rangle$  qubit state. Part (2) has an anti control-control Hadamard which eliminates  $|011\rangle$  state. Part (3) has an anti control-anti control Hadamard gate preceded and succeeded by a Pauli-X in the third qubit to eliminate the  $|000\rangle$  state. The circuits of the first 4 models will have a Hadamard initially fixed on its third qubit to ensure that it generates scores from 1 to 5.

# A. Model with fixed target and bow position, no user-interaction and ideal conditions.

Our initial circuit for this model is depicted in Fig. 2. To this, we add Hadamard gates to the first and second qubit to create 8 superposed states which have equal

![](_page_61_Figure_0.jpeg)

FIG. 7: Quantum circuit for III B. The circuit is same as that of the probabilistic case III A, the only difference being that two  $R_y$  gates are added to simulate physical conditions which alters the probabilities depending on the value of  $\theta$ .

![](_page_61_Figure_2.jpeg)

FIG. 8: Final Probabilities of III B for  $\theta = 5pi/3$ 

![](_page_61_Figure_4.jpeg)

FIG. 9: Final Probabilities of III B for  $\theta = \pi/12$ 

![](_page_61_Figure_6.jpeg)

FIG. 10: Circuit for III C: The qin1 and qin2 qubits are for user to input his position and the  $R_y$  gates simulate physical conditions.

probabilities (Fig. 3). The rest of the circuit eliminates the unnecessary states and then the final probabilities are evaluated. As seen in Fig. 5, we find out that the middle scores of 2,3 and 4 have probabilities ranging from 24-30, significantly more than the extreme scores of 1 and 5 which have probabilities in the range 12-15. This makes sense as the user i.e., the computer shoots arrows randomly in ideal conditions, therefore its probability of hitting middle scores is more than that of extreme stores.

# B. Model with fixed target and bow position, no user-interaction but varying physical conditions.

The idea of physical condition is such that it alters probabilities of 1 to 5 based on some given parameter. Also, the eliminated states should not come back. Therefore, the  $R_y$  gate serves our purpose perfectly, which rotates the states around y-axis and changes their probabilities but does not allow the inclusion of a complex part. In the circuit of Model 1 (Fig. 3), we add  $R_y$  gates after the Hadamard ones in the first and second qubit (Fig. 6). Thus, our probabilities vary depending on the value of the  $\theta$  assumed by  $R_y$  gate. Now, we may code a program where the value of  $\theta$  is generated randomly and therefore the user's (the computer) probability of hitting different points on the circle change in every turn. We see in the Fig. 8 and Fig. 9 that how different the probabilities are for two different values of  $\theta$ .

## C. User-interactive model with fixed target and changeable bow position according to variable physical condition.

For this model, we keep the Hadamard gate fixed on the third qubit. We take two qubits qin1 and qin2 for the user to input in the first and second qubit respectively. The user has 4 possible input combinations:  $|00\rangle$ ,  $|10\rangle$ ,  $|01\rangle$  and  $|11\rangle$ . The qubits we work with (q[0] and q[1]) are updated according to our input qubits. We use  $R_y$  gate to simulate physical conditions (Fig. 10) where the pa-

![](_page_62_Figure_0.jpeg)

FIG. 11: Quantum circuit for III C. The qubits q[5] and q[6] are the input qubits, the qubits that we work with are updated using the CNOT gate. The rest of the circuit remains the same with  $R_y$  gates simulating the physical conditions.

![](_page_62_Figure_2.jpeg)

FIG. 12: Final Probabilities of III C for input  $|00\rangle$ at  $\theta = 5\pi/3$ 

![](_page_62_Figure_4.jpeg)

FIG. 13: Final Probabilities of III C for input  $|10\rangle$ at  $\theta = 5\pi/3$ 

![](_page_62_Figure_6.jpeg)

FIG. 14: Final Probabilities of III C for input  $|01\rangle$ at  $\theta = 5\pi/3$ 

![](_page_62_Figure_8.jpeg)

FIG. 15: Final Probabilities of III C for input  $|11\rangle$ at  $\theta = 5\pi/3$ 

![](_page_62_Figure_10.jpeg)

FIG. 16: Circuit for III D: The qubits qin1 and qin2 are for the user to input the qubit combination. The qubit qin3 is for the user to trigger the special biasing, whereas the ancilla qubit  $|0\rangle$  is used to check whether the second qubit of the input is 0 or not.

rameter  $\theta$  is randomly distributed but the coded program constricts it in the range  $[0,2\pi]$ . It is assigned a value at the beginning of the game using the statement:  $\theta = 2\pi^*$ random.random() where random is a function of random module in python. From the Table I, we can see that in the range  $\theta \epsilon(\pi/2, 3\pi/2)$ , the input of  $|11\rangle$  has the highest probability of getting 4, whereas the input of  $|01\rangle$  has the highest probability of 5 but an equally high chance of 1. Similarly, in the intervals  $(0, \pi/2) \cup (3\pi/2, 2\pi), |00\rangle$ input has the highest probability of 4 and  $|10\rangle$  has highest probability of 5 but an equally high probability of 1. Exploiting the pattern of possibilities obtained (Table I), we can design the rules of the game. The user must achieve a target score in a specified number of turns and for every turn a random value of  $\theta$  is generated and its range is checked, the user is given the information about the range (i.e., the prevailing physical conditions) and according to the rules, he chooses his input qubits so as to achieve his target score in the number of turns provided. Since the value of  $\theta$  varies in every turn, the user's input combination does so too, according to which the circuit gets updated each time. Now, the user may keep inputting a sequence so as to get 4 or 5 as his score, but as the level gets harder, we may keep him restricted by making it a rule that he gets eliminated if his final score

![](_page_63_Figure_0.jpeg)

FIG. 17: Quantum circuit for III D. The qubit q[4] is for the user to trigger the special biasing. An input of  $|1\rangle$  triggers the biasing and an input of  $|0\rangle$  does otherwise. The qubits q[9] and q[10] are the input qubits, the value of which update the qubits that we work with. Extra qubits q[5] and q[6] are taken since we have a

control-control U3/X gate but only a single control U3 gate and a double control X is provided in IBM QE. q[7] checks whether the second qubit of the input combination is 0 or not. Again, extra qubit q[8] is taken since we have a control-control-control-control U3/H gate and IBM QE provides only a single control gate.

![](_page_63_Figure_3.jpeg)

FIG. 18: Final probabilities of III D with biasing triggered, input  $|00\rangle$  and  $\theta=5\pi/3$ . On comparing with the unbiased probabilities of Fig. 12, we find that all of 2's probability has been shifted to 4 and a huge portion of 1's probability has been shifted to 5.

crosses the target score by a certain limit (this limit gets more and more constricted as levels get harder). But since we are working with a 5-pointer target, we can design a maximum of 3 or 4 levels. Figs. 12, 13, 13 and 15 show the variation in probabilities for the 4 different inputs for  $\theta=5\pi/3$ .

# D. User-interactive model with fixed target, special bow and arrow option and also, changeable bow position according to variable physical condition.

To make this game interesting we add a special feature: a special bow and arrow. To implement this concept, we take an extra qubit (qin3), inputting  $|1\rangle$  in which triggers the special bow and arrow concept. Now, by the table given, we know about the pattern of probabilities obtained for the various combination of inputs and value of  $\theta$  (of  $R_y$  gate). Now, what the special bow and arrow does is that it reduces the probability of getting 1 by some extent and increases that of getting 5 by the same extent. Similarly, it reduces the probability of getting 2 by some extent and increases that of getting 4 by the same extent. The probability of getting 3 remains same throughout. The special bow and arrow works on every combination (of input and  $\theta$  of  $R_{u}$ ) differently but carries out the same function. But to avail this advantage, the user pays a price i.e., his initial score is in negative and not zero (which increases as levels get harder).

To decrease probability of 1 and increase that of 5, we

use control-anti-control-gate to apply a U3 on the third qubit with  $\theta = \pi/3$  (this value simplifies the matrix). To decrease probability of 2 and increase that of 4, we first check if the second entry of our input combination (as in the bow and arrow position) is zero or not. If it is zero, we flip the ancilla qubit by applying an X gate on q[1]. After that we apply an anti-control-anti-control-control U3 gate with  $\theta = \pi/3$  on third the third qubit (q[2]) (which works if the second bit of the input combination is 0) followed by an anti-control-anti-controlcontrol Hadamard gate on the third qubit (which works if the second bit of the input combination is 1) (Fig. 16). In the beginning, the user is given a choice of special bow and arrow  $(|1\rangle$  to avail it and  $|0\rangle$  to not avail it). The rest of the game remains the same as in IIIC where the user chooses his bow position after being informed about the value of  $\theta$  assumed by  $R_y$  gate. We study the changes brought about in biasing in Fig. 18.

# E. Model with constantly moving target, no user-interaction and ideal conditions.

We, now propose a simple mathematical model for a moving target. We consider that the user is in a fixed position and shooting the arrows in a fixed direction whereas the target is oscillating between two fixed points. In this case, our possible scores range from 0 to 7. Now we can easily visualize that the probability of getting these scores must follow the pattern: i.e., the probability of getting 0,1,2,3 should simultaneously increase and probability of getting 4,5,6 and 7 should simultaneously decrease with changing time/phase. After a certain time/phase, the probability of getting 0,1,2 and 3 decreases and that of getting 4.5,6 and 7 increases simultaneously. We realize that this can be achieved if we shift the phase of 4,5,6 and 7 by  $\pi/2$ . Hence, we develop the following position operator ( $\hat{U}_x$ ):

![](_page_64_Figure_0.jpeg)

FIG. 19: Circuit for III E: The three  $R_y$  gates are used to initialize a state. The ancilla qubit  $|0\rangle$  taken is necessary to apply the gates according to the position matrix formulated. The QFT and inverse QFT are applied to create momentum operator and alter probabilities.

|--|--|--|--|--|--|

FIG. 20: Quantum circuit for III E. The three initial  $R_y$  are used to initialize the states. The first part of the circuit implements the quantum-Fourier transformation for the 3 qubit system. Then we apply the gates according to the position operator matrix formulated. The anti-control qubit is made by preceding and succeeding the control qubit by X gate. The extra qubits q[3] and q[4] are taken since we have a control-control-control-control U1 gate but IBM QE provides a single control U1. We finish the circuit by implementing an inverse QFT.

![](_page_64_Figure_4.jpeg)

FIG. 21: Probabilities for the moving target model. The (a) part corresponds to the initial state whereas (b), (c), (d) correspond to the probabilities as we vary  $\phi$  from 0 to  $\pi/4$ . As we can see, the probabilities of 0,1,2,3 decrease whereas the probabilities of 4,5,6,7 increase at the same time.

| [1 | 0                       | 0               | 0                       | 0               | 0                       | 0               | 0                       |
|----|-------------------------|-----------------|-------------------------|-----------------|-------------------------|-----------------|-------------------------|
| 0  | $e^{\iota(\phi-\pi/2)}$ | 0               | 0                       | 0               | 0                       | 0               | 0                       |
| 0  | 0                       | $e^{\iota\phi}$ | 0                       | 0               | 0                       | 0               | 0                       |
| 0  | 0                       | 0               | $e^{\iota(\phi-\pi/2)}$ | 0               | 0                       | 0               | 0                       |
| 0  | 0                       | 0               | 0                       | $e^{\iota\phi}$ | 0                       | 0               | 0                       |
| 0  | 0                       | 0               | 0                       | 0               | $e^{\iota(\phi-\pi/2)}$ | 0               | 0                       |
| 0  | 0                       | 0               | 0                       | 0               | 0                       | $e^{\iota\phi}$ | 0                       |
| 0  | 0                       | 0               | 0                       | 0               | 0                       | 0               | $e^{\iota(\phi-\pi/2)}$ |
| -  |                         |                 |                         |                 |                         |                 | (1)                     |

To create the momentum operator, we apply the quantum Fourier transformation before the position operator and an inverse QFT after the position operator  $\hat{U}_p = [F^d]$  $\hat{U}_x \ [F^d]^{-1}$ . This momentum operator serves our purpose and we can design a quantum circuit that simulates a harmonically moving target [15] (Fig. 19).

![](_page_64_Picture_8.jpeg)

FIG. 22: **QR** code for acknowledgements, bibliography and appendix

# IV. CONCLUSION

In this article, we have proposed five simple mathematical models and corresponding quantum circuits for the game of archery. Deriving inspiration from these circuits, we can further create different models of the game with even more complexities and increased user interaction. For example, we can make user-interactive models for moving target where the user gets to place his bow and arrow in accordance to the instantaneous position of the target. This suggested model will require a system with more number of qubits for user input and more intricate designing of gates for a strong mathematical algorithm. Similarly, we can vary the distance of the player from the target and impose more challenging physical conditions with the help of detailed mathematical logic. Also, we can figure out the algorithm for a 10-pointer target using a higher number of qubits. Hence, our aim is satisfactorily fulfilled as we have successfully put forward a model serving as a template for future extrapolation and game development.

The acknowledgement, bibliography and appendix are contained in the qr code given (Fig. 22).

# SCENE 6:

![](_page_65_Picture_1.jpeg)

![](_page_65_Picture_2.jpeg)

![](_page_65_Picture_3.jpeg)

MEME CREDITS: NEEL LOHIT DASH SWAPNILA CHAKRABARTY

# INTERACTION OF A PLANET WITH A BINARY-STAR SYSTEM

# MEMBERS

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(2<sup>nd</sup> PHYSICS)

# 1. INTRODUCTION

Stellar dynamics is the field of study where we examine the effects of bodies of more or less equal masses or contributions of gravity and their evolution. In this article, we provide a short summary of the report of the problem related to orbits of a simple binary star system solved numerically. This is a simplified version of the three-body system where we take two of the stars to be massive at a fixed distance. We had taken various special situations only to slowly increase the generality of the problem by including the effect of the planet's mass on the motion of the stars and study various orbits and their stability.

# 1.1 THE PROBLEM

- 1. Keeping both stars fixed in place, with M1=M2= 1 and the planet at  $(x_0, y_0) = (0.4, 0.5)$  with  $(V_{x_0}, V_{y_0}) = (0, -1)$ . (Where M<sub>1</sub>, M<sub>2</sub> is mass of the Star/Sun 1 and 2respectively, x and y are their respective position and V is the velocity.) Observe the kind of orbit formed.
- 2. Keeping masses equal, vary the initial conditions of the planet and observe the orbits. Find stable orbits of various types.
- 3. Make one star twice as massive as the other one and observe the changes that happen to the orbit when we use the initial conditions for stable orbits obtained in the previous case. Change the initial conditions to find a stable orbit for this new system.
- 4. Keeping the masses equal, allow the stars to move to find the type of stable orbits this binary system can have. Take necessary assumptions required to ignore the effect of the planet on the orbit of the stars. Once a stable orbit is achieved for the Star system, vary initial conditions of the planet to obtain stable orbits for the planet.
- 5. Keeping the masses equal and allowing the stars to move, observe the orbit of the stars while considering the effect of the planet's mass on the system. Take an appropriate ratio for the star vs planet and find stable orbits for this system.
- 6. Vary mass of the two stars and again study orbits with and without the influence of planet.
- 7. Do the problem for the non-fixed star cases using a non-inertial frame, such as one of the stars itself so that it appears to be stationary. Study how the orbits appear in this frame.

# 2. ASSUMPTIONS

- Our 'binary-stars with the planet' system is isolated in the universe and no external force is acting upon it. It evolves with time only due to its internal interactions. This is a valid approximation of real systems, considering the fact that every other significant (massive) body in the universe resides at length scales far greater than our system's natural length scale.
- Newton's laws and laws of classical mechanics holds true for the system at all points of time. This means that special relativistic effects are ignored. So, to have sensible results, it is implicitly assumed

that velocity of each body concerned is much lower than the speed of light. Also, the precision we can work with is such that general relativistic effects are of no importance.

The following are the case-specific assumptions and simplifications that we used in the study.

# 2.1 BOTH STARS FIXED

We start by taking a very crude assumption of keeping both stars fixed at a particular distance from each other. Although two stars in the centre of mass frame cannot remain fixed with respect to each other without any external force keeping them fixed. The existence of this external force then should also influence the planet. But to get started, we ignore these complexities and assume that somehow the stars are fixed and the planet moves due to the gravitational pull of both the stars.

# 2.2 STARS NOT FIXED

# 2.2.1 IGNORING PLANET'S INFLUENCE

The position and velocity of centre of mass of the system has no dependence on the planet. The contribution of one star to the other's acceleration is far greater than the contribution of the planet, so the latter can be ignored in calculations.

# 2.2.2 INCLUDING PLANET'S INFLUENCE

In this case, no assumptions are taken except the general ones.

# 3. THE NON-DIMENSIONALISED EQUATIONS

We begin by choosing a suitable coordinate system. Rectangular coordinates system has been used in each case since it is more suited for computational analysis than polar coordinates which suits analytic solving.

# 3.1 FIXED STARS INCLUDING DIFFERENT MASS RATIO

Based on our system a natural length scale is  $l_0$ . Hence, we begin non-dimensionalising the equations just obtained. In units of lo the position coordinates will become :  $x = l_0 \times X$  and  $y = l_0 \times Y$ .

# Equation of planet:

$$\frac{d^2X}{d\tau^2} = -4\left[\frac{(2X+1)}{[(1+2X)^2 + (2Y)^2]^{3/2}} + \frac{r(2X-1)}{[(2X-1)^2 + (2Y)^2]^{3/2}}\right]$$

$$\frac{d^2Y}{d\tau^2} = -8\left[\frac{Y}{[(1+2X)^2 + (2Y)^2]^{3/2}} + \frac{rY}{[(2X-1)^2 + (2Y)^2]^{3/2}}\right]$$

# 3.2 STARS NOT FIXED INCLUDING DIFFERENT MASS RATIO

# 3.2.1 IGNORING PLANET'S INFLUENCE

# 3.2.1.1 CENTRE OF MASS FRAME

Again, by taking the natural units lo as initial distance between the stars and  $T_0 = \sqrt{(l_0)^3/GM_1}$  and converting dimensional position and time into these units.

For planet:

$$\begin{aligned} \frac{d^2 X}{d\tau^2} &= \left[ \frac{-(X+rX_2)}{[(rX_2+X)^2+(rY_2+Y)]^{3/2}} + \frac{r(X_2-X)}{[(X_2-X)^2+(Y_2-Y)^2]^{3/2}} \right] \\ \frac{d^2 Y}{d\tau^2} &= \left[ \frac{-(Y+rY_2)}{[(rX_2+X)^2+(rY_2+Y)]^{3/2}} + \frac{r(Y_2-Y)}{[(X_2-X)^2+(Y_2-Y)^2]^{3/2}} \right] \end{aligned}$$

For star 2:

$$\frac{d^2 X_2}{d\tau^2} = -\left[\frac{(1+r)X_2}{[((1+r)X_2)^2 + ((1+r)Y_2)^2]^{3/2}}\right]$$
$$\frac{d^2 Y_2}{d\tau^2} = -\left[\frac{(1+r)Y_2}{[((1+r)X_2)^2 + ((1+r)Y_2)^2]^{3/2}}\right]$$

# 3.2.1.2 FRAME OF REFERENCE OF STAR 1

For star:

$$\frac{d^2 X_2}{d\tau^2} = -\frac{(1+r)X_2}{(X_2^2 + Y_2^2)^{3/2}}$$

$$\frac{d^2 Y_2}{d\tau^2} = -\frac{(1+r)Y_2}{(X_2^2 + Y_2^2)^{3/2}}$$

For planet:

$$\frac{d^2 X_2}{d\tau^2} = -\frac{X}{(X^2 + Y^2)^{3/2}} - \frac{r(X - X_2)}{[(X - X_2)^2 + (Y - Y_2)^2]^{3/2}} - \frac{X_2}{(X_2^2 + Y_2^2)^{3/2}}$$

$$\frac{d^2 Y_2}{d\tau^2} = -\frac{Y}{(X^2 + Y^2)^{3/2}} - \frac{r(Y - Y_2)}{[(X - X_2)^2 + (Y - Y_2)^2]^{3/2}} - \frac{Y_2}{(X_2^2 + Y_2^2)^{3/2}}$$

# 3.2.2 INCLUDING PLANET'S INFLUENCE

Natural length scale lo is the initial distance between the stars. We choose a different Natural time scale  $T_0: \frac{1}{T_0^2} = \frac{GM_2}{l_0^3}$  for mathematical simplification.

For star 1:

$$\begin{aligned} \frac{d^2 X_1}{d\tau^2} &= \left[ \frac{X_2 - X_1}{[(X_2 - X_1)^2 + (Y_2 - Y_1)^2]^{3/2}} - \frac{r_2^3 [(r' + r_2)X_1 + X_2]}{[((r' + r_2)X_1 + X_2)^2 + ((r' + r_2)Y_1 + y_2)^2]^{3/2}} \right] \\ \frac{d^2 Y_1}{d\tau^2} &= \left[ \frac{Y_2 - Y_1}{[(X_2 - X_1)^2 + (Y_2 - Y_1)^2]^{3/2}} - \frac{r_2^3 [(r' + r_2)Y_1 + Y_2]}{[((r' + r_2)X_1 + X_2)^2 + ((r' + r_2)Y_1 + y_2)^2]^{3/2}} \right] \end{aligned}$$

For star 2:

$$\begin{aligned} \frac{d^2 X_2}{d\tau^2} &= \left[ \frac{r'(X_1 - X_2)}{[(X_1 - X_2)^2 + (Y_1 - Y_2)^2]^{3/2}} - \frac{r_2^3 [(r'X_1 + (r_2 + 1)X_2]}{[(r'X_1 + (1 + r_2)X_2)^2 + (r'Y_1 + (1 + r_2)Y_2)^2]^{3/2}} \right] \\ \frac{d^2 Y_2}{d\tau^2} &= \left[ \frac{r'(Y_1 - Y_2)}{[(X_1 - X_2)^2 + (Y_1 - Y_2)^2]^{3/2}} - \frac{r_2^3 [(r'Y_1 + (r_2 + 1)Y_2]}{[(r'X_1 + (1 + r_2)X_2)^2 + (r'_1Y_1 + (1 + r_2)Y_2)^2]^{3/2}} \right] \end{aligned}$$

# SOME OF THE RESULTS WE OBTAINED

![](_page_69_Figure_8.jpeg)

![](_page_70_Figure_0.jpeg)

Orbits as seen from the centre of mass  $X = 4.0 \ Y = 3.5$  (b) Orbits as seen from starl X = 4.5,  $Y = 3.5 \ V_X = -0.6 \ V_X = -0.6 \ V_Y = 0.5 \ X_2 = 0.5 \ Y_2 = 0. \ V_{X_A} = 0. \ V_{Y_A} = 0.5 \ V_Y = 1.0 \ X_2 = 1.0 \ Y_2 = 0. \ V_{X_A} = 0. \ V_{Y_A} = 1.0 \ Y_A = 0.0 \$ 

03

0.2

01

≶ 00

-0.1

-0.2

-0.3

-10

(b) Gradually varying asymmetrical phase plot

2

0

-1

coordinate

ź

-5

-15

![](_page_70_Figure_2.jpeg)

(a) Orbit circular but stars not at centre.

![](_page_70_Figure_4.jpeg)

![](_page_70_Figure_5.jpeg)

Phase plot of planet

o x

10 15 20

ORBIT

2 3 4

(a)  $\Delta au = 0.01$ ,  $au_f = 1000$ 

(b)  $\Delta \tau = 0.001$ ,  $\tau_f = 1000$ 

**Fig:** Orbit Doesn't change on increasing precision. Implies less sensitive to minute changes in position and velocities. X1 = 0.00990097 Y1 = 0.VX1 = 0.VY1 = 0.1 X2 = X1-1 Y2 = 0.VX2 = 0.VY2 = -9.9999925

# CONCLUSION

There can be several different types of stable binary-star and planet systems.

if planet mass is very small, it can only exist in a very large orbit. This is not feasible in practice since there would definitely be other significant bodies inside the orbit of planet and the star that will influence the planet's orbit. Hence, it's an idealistic situation.

So, we focus on compact orbits.

For ratios 10^5-8, planet moves in orbits comparable to orbits of the stars.

When both stars are almost equally massive, they move in elliptical and overlapping orbits.

Perfect circular orbits, that exactly superimpose on each other are not possible unless we ignore the influence of planet in the COM, since these orbits would mean that COM is fixed at the centre and hence any movement of the planet is not happening.

In the former case, the system is more resistant to changes since we see the deviations that occur with each revolution manifested in the increasing thickness of the plotted trajectory curves. Yet both planet and stars remain in orbit for long times.

With unequal masses i.e. one star being much massive (100,1000 times or more), the possibility of stable compact systems is much more.

To find the full report that includes discussions, interpretations, methodology etc, please scan the following QR code:

![](_page_71_Picture_10.jpeg)
# SCENE 7:



PEACE ON EARTH

DOODLE CREDITS: DITHAINGAM PANMEI

#### **Analysis of the 3-Body Problem**

#### - Vedanta Thapar and Hariprasad SV, 1st Physics

In the late 17th Century, Sir Isaac Newton published his 'Principia Mathematica'. In this revolutionary work he outlined his 3 Laws of Motion and his Universal Law of Gravitation as well as some of their implications. With this mathematical framework came the 3-body problem.

The 3 Body Problem is a subset of a much larger idea of n-body problems. It is a historic problem that aims to solve for the equations of motions of 3 bodies in 3-dimensional space, with only their mutual gravitational attraction, given their initial positions and momenta. Over the years several physicists and mathematicians have tried their hand at solving this problem however we still have no useful general analytical solution. Now it is hard to understate the importance of this problem and of other n-body problems: for example, our very own solar system is an n-body problem and parts of it can be approximated to a 3-body system. So, in the universe we often see 3-body systems which are largely isolated (a binary star system with a planet for example) hence it is very important to try and find a useful analytical solution for it.

The equations of Motion for a 3 body System look like:

$$\frac{d^{2}\vec{r_{1}}}{dt^{2}} = -G\left(\frac{M_{2}}{|\vec{r_{1}} - \vec{r_{2}}|^{3}}(\vec{r_{1}} - \vec{r_{2}}) + \frac{M_{3}}{|\vec{r_{1}} - \vec{r_{3}}|^{3}}(\vec{r_{1}} - \vec{r_{3}})\right)$$

$$\frac{d^{2}\vec{r_{2}}}{dt^{2}} = -G\left(\frac{M_{1}}{|\vec{r_{2}} - \vec{r_{1}}|^{3}}(\vec{r_{2}} - \vec{r_{1}}) + \frac{M_{3}}{|\vec{r_{2}} - \vec{r_{3}}|^{3}}(\vec{r_{2}} - \vec{r_{3}})\right)$$

$$\frac{d^{2}\vec{r_{3}}}{dt^{2}} = -G\left(\frac{M_{1}}{|\vec{r_{3}} - \vec{r_{1}}|^{3}}(\vec{r_{3}} - \vec{r_{1}}) + \frac{M_{2}}{|\vec{r_{3}} - \vec{r_{2}}|^{3}}(\vec{r_{3}} - \vec{r_{2}})\right)$$

#### So, what makes it so difficult to solve this problem?

Now Newtonian Physics is essentially deterministic, i.e., given the initial conditions of any body (its momentum, position, forces acting on it, etc.) we can apply Newton's laws and solve for its motion at some later time exactly; now this is true at least in principle. However, the real world is far more complicated.

The 3-body system is what we call a chaotic system:

- The system is very sensitive to initial conditions: small changes in the initial conditions can lead to hugely different orbits
- It tends to chaos: the motion of the bodies loses any discernible pattern.

One of the big issues is when we say 'given the initial conditions'. Theoretically, there's no limit to the accuracy for which these initial conditions depend, however, in a practical sense there is only a certain degree of accuracy up-to which we can measure the initial conditions. Hence systems that are very sensitive to these conditions tend to be incredibly difficult to solve analytically.



#### The Different Approaches over the Years

Over the last 300 years, several mathematicians and physicists have tried their hand at this problem. While we still haven't achieved our final objective, we do have several approximate and special solutions to the problem.

#### • Approximate Solutions: The Restricted 3 Body Problem

The restricted 3 body problem is the most applied in the real world specially in space travel and the analysis of the motion of satellites. The condition that we apply here is that one of the masses is much lighter than the rest i.e. the larger bodies will not feel its gravitational influence however, it will feel theirs. This is analogous to the idea of a test charge used in electrostatics to define Potential and Electric Field.

For 3 masses:  $M_1, M_2, M_3$  where  $M_3 \ll M_1$  and  $M_3 \ll M_2$ .

Now since this system is isolated, its Centre of Mass must move with a uniform velocity i.e. it can act as an inertial frame of reference.

 $\Rightarrow \text{Taking COM as origin let } \overrightarrow{r_1} \rightarrow Position \ vector \ of \ Mass \ M_1 \ w.r.t. \ COM \\ \overrightarrow{r_2} \rightarrow Position \ vector \ of \ Mass \ M_2 \ w.r.t. \ COM \\ \overrightarrow{r_3} \rightarrow Position \ vector \ of \ Mass \ M_3 \ w.r.t. \ COM \end{cases}$ 

Hence, we can write the relative positions as:  $\vec{r_{12}} = \vec{r_1} - \vec{r_2}$  and similarly we can define  $\vec{r_{23}}$  and  $\vec{r_{13}}$ .

For this case Newton's Second Law can be approximated as:

$$\frac{d^{2}\vec{r_{1}}}{dt^{2}} = -G\frac{M_{2}}{|\vec{r_{12}}|^{3}}\vec{r_{12}}$$
$$\frac{d^{2}\vec{r_{2}}}{dt^{2}} = G\frac{M_{1}}{|\vec{r_{12}}|^{3}}\vec{r_{12}}$$
$$\frac{d^{2}\vec{r_{3}}}{dt^{2}} = G\left(\frac{M_{1}}{|\vec{r_{13}}|^{3}}\vec{r_{13}} + \frac{M_{2}}{|\vec{r_{23}}|^{3}}\vec{r_{23}}\right)$$

The reason this restricted problem can be solved is that the motion of the 2 large masses is unaffected by the 3rd mass. Hence, they can be approximated as a 2-body system for which we have an exact solution. Furthermore, this implies that their motion is planar hence this reduced the number of effective coordinates in the problem.

Another important form of the restricted 3 body problem that is often applied in nature is when the distances between the bodies is large. Here, again we can consider 2 bodies and ignore the effect of the 3rd forming an approximate 2 body system.

#### • Special Cases:

There have been several solutions for special initial conditions but the most famous are Euler and Lagrange Solutions. They both solved the 3-body problem for 2 special initial conditions:

**Euler:** He solved this problem for the special case when all 3 bodies revolve around a mutual center of mass in a straight line. However, it is not a very stable orbit.



**Lagrange:** He solved this problem for the condition that the 3 bodies form an equilateral triangle. These orbits are stable under special conditions (when one mass is much greater than the others) but like almost all 3 body systems, it tends to chaos in longer time scales. A simulation of Lagrange's Case can be found by scanning the QR code at the end of this article.

#### • Karl Sundman's Solution:

After working on the problem for many years, famous Mathematician Henri Poincare announced that this problem cannot be solved, no analytical solution exists. A few years after Poincare's bold claim, Karl Sundman refuted his statement and actually provided a completely general and analytical solution to the problem. But there was a catch:

The solution was in the form of an infinite convergent series that converges so slowly that it would need at least  $1 \times 10^{14}$  terms before we could make any useful predictions from it. Hence it still remains without a useful general analytical solution.

$$x(t) = \sum_{k} a_k t^{2k/3}$$

#### • Periodic Solutions:

Each orbit for a system of 3 bodies is specified uniquely by the initial conditions leading to it. Some extremely specific initial conditions lead to very stable systems which are the Periodic Solutions to this historic problem.

They were first computed by Richard Montgomery and Alain Chenciner in a paper published in the Year 2000: they approached the problem through a topological perspective by assuming the three massive bodies on the vertices of a triangle whose angles are mapped over the surface of a sphere. This

#### A remarkable periodic solution of the three-body problem in the case of equal masses

By ALAIN CHENCINER and RICHARD MONTGOMERY

helped them compute the initial conditions to a number of periodic systems.

One of the most unexpected and beautiful periodic orbits is the Figure 8 Solution for Equal

**masses.** The method for finding this solution follows the '*Principle of Least Action*': this involves minimizing the path the orbit takes. However, this method generally leads us to chaotic orbits. Additionally, this orbit is incredibly unexpected since most equal mass systems tend to be more unstable. These initial conditions were numerically computed by a Spanish mathematician Carles Simo in 2000.



Till date researchers have been computing more initial conditions that lead to such orbits: in 2017 researchers Xiaoming Li and Shijun Liao reported 669 new periodic orbits of the equalmass zero-angular-momentum three-body problem. This was followed in 2018 by an additional 1223 new solutions for a zero-momentum system of unequal masses.

#### Numerical Approach

With the onset of computers numerical solutions became more refined and could be applied to a large range of problems for which it was too difficult to solve analytically. The idea we apply is that we take some small interval of time and we update our positions, velocities and accelerations for each time step approximating that they don't change over that small interval. One of the more refined algorithms for doing this is the **Verlet Algorithm** which looks like:

Now since we're dealing with vectors we have to solve separately for each direction:

For x direction:

$$x(t+\frac{\Delta t}{2})=x(t)+\frac{\Delta t}{2}v(t)$$

Using this new  $x(t + \frac{\Delta t}{2})$  we calculate the new relative displacements and from there calculate new accelerations :  $a(t + \frac{\Delta t}{2})$  using this we perform the next step of the Verlet Algorithm:

$$v(t+\Delta t)=v(t)+a(t+rac{\Delta t}{2})\Delta t$$

Now using  $v(t + \Delta t)$  we will perform the last step of the algorithm:

$$x(t + \Delta t) = x(t + \frac{\Delta t}{2}) + v(t + \Delta t)\frac{\Delta t}{2}$$

#### Here, $\Delta t \rightarrow small time interval$

The Python Code we wrote for solving this numerically can be found by scanning the QR code at the end of the article.

#### • Sample Trajectories

We have used the module *'vpython'* to produce some simulations for different cases of the 3-body system:

1. Here, we have a system which shows us what happens with most 3 body systems. Often times to gain stability one body gets ejected from the system and the remaining 2 form a stable 2 body system.



One body gets ejected while other two form a stable system

2. Here we have 2 images, these trajectories are for the Sun, Earth and Venus but they have been approximated to circular orbits:





Now, we have an orbit over a much larger time scale:



Clearly, we still see deviations from the circular path over a very large period of time

#### • Conclusion

Being such a significant and unsolved problem, the 3-body problem as well as other n-body systems are still a field of active research. In September 2019, scientists Nicholas C. Stone and Nathan W.C. Leigh suggested a new way to solve the problem: they came at it with a new statistical approach that involved calculating the probability of the orbits and using that to make sense of the chaos. However, this new tool as well as others are still an active area of research considering their great importance in the real world.

For extra resources including simulations and the code that we have made, please scan the QR code:





Recently, on 11th February, 2021 we celebrated the 6th International Day of Women and Girls in Science. The fight for 'equal rights' is at its peak around the world yet women are severely under-represented in science, especially physics. But despite all the obstacles, some women take up the challenge, overcome them and then serve as inspirations to the other girls who dream of a cosmos fairytale. We take this opportunity to celebrate Andrea Ghez, who in 2020 became only the fourth woman to receive a Nobel Prize in Physics. And then we revisit the fiercest woman, rather person, science has ever known, Marie Curie.



In 2020, the Nobel Prize in Physics was awarded to 3 scientists: Roger Penrose for predicting the existence of Black Holes as a result of General Relativity, Andrea Ghez and Reinhard Genzel "for the discovery of a supermassive compact object at the centre of our galaxy." The only existing explanation of what this object could be is a supermassive black hole. This object was named Sgr A\* and today the centres of most galaxies are believed to be similar supermassive black holes. Ghez and Genzel both worked independently on different hemispheres of the planet and proved what was believed to be a pointless endeavour: primarily constrained due to lack of instrumentation.

#### **Andrea Ghez's Research**

#### So how do you find something you cannot see.....

Well the idea is that we observe its effects on the matter around it and use that to actually locate it or prove it exists. Any object whose mass is confined within the 'Schwarzschild radius' associated with that mass can be proved to be a black hole (at least from the physics we know of). So the method that Ghez used to actually find this invisible object involved using the surrounding matter to predict the mass of the object at the centre and finding its radius. If the mass is confined within the Schwarzschild radius then it is a black hole.

$$r_s=rac{2GM}{c^2}$$

Now actually proving this for the centre of our galaxy reuired a large amount of observation time in order to see how the stars and dust around the object at the centre behave. She essentially had to calculate how fast these stars orbit (this calculates the mass) and the scale of the orbit(this calculates the ). This ruled out the use of orbiting telescopes and as a result they had to resort to terrestrial telescopes which is far more difficult due to light and atmospheric pollution. Furthermore analyzing the stars' orbits involved taking in a huge amount of de tail.She essentially had to calculate how fast these stars orbit (this calculates the mass) and the scale of the orbit(this calculates the ). This ruled out the use of orbiting telescopes and as a result they fast these stars orbit (this calculates the mass) and the scale of the orbit(this calculates the ). This ruled out the use of orbiting telescopes and as a result they had to resort to terrestrial telescopes which is far more difficult due to light and atmospheric pollution. Furthermore analyzing the stars' orbits involved taking in a huge amount of de tail. She essentially had to calculate how fast these stars orbit (this calculates the mass) and the scale of the orbit(this calculates the ). This ruled out the use of orbiting telescopes and as a result they had to resort to terrestrial telescopes which is far more difficult due to light and atmospheric pollution. Furthermore analyzing the stars' orbits involved taking in a huge amount of detail.

Andrea worked at the W.M. Keck Observatory's twin telescopes on Mauna kea, Hawai. She started out on the Near Infrared Camera(NIRC: an instrument that was never designed for such a task) and she needed an ultrafast readout of images and then a restacking of the result to remove the effects of the atmosphere's turbulence. After difficult months of observations she and her team found the first hints of stars orbiting around something at the centre.

Around the same time a new technology had just been developed: Adaptive Optics. Adaptive optics basically accounts(in real time) for atmospheric disturbances that can distort the incoming wavefront by deforming the mirror to compensate for the distortion. Ghez ended up as one of the pioneers of this technology and used it in her observations of the centre of our galaxy.

In total Ghez studied the orbits of more than 3000 stars about the supposed centre and made measurements that suggested the existence of a supermassive(hence gravitationally powerful) yet compact object. This black hole has now served as a close laboratory to see the gravity around such massive black holes: it can act as a test for General Relativity in a strong gravity environment and it even brought up a number of new questions in astrophysics related to the age of the stars orbiting the black hole. Overall this groundbreaking discovery is another step in our quest to understand the formation and evolution of galaxies and our universe as a whole.

-Authored by Vedanta Thapar, 1st Phy

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https://arxiv.org/pdf/1207.6755.pdf The Paper that Andrea and her team published in 2009 https://astronomy.swin.edu.au/cosmos/S/Schwarzschild+Radius#:~:text=The%20Schwarzschild%20ra dius%20is%20the,will%20be%20a%20black%20hole.

# Marie Gurie

A passage from the Kannada drama Belakinondu Kirana by Dr.Prsad Rao based on the book of the same name by Nemichandra in the form of soliloquy of Madame Curie, [translated by Shalika Yekkar, 2nd Physics]

My beloved Irene, the moment my eyes glanced through the words in your letter and my brain comprehended it, a rush of emotions ran through me. I felt myself sinking in an urge to hug you,to hold you like you were still a newborn in my arms. Here in France, things around cannot be labelled as 'fine'. They say, to survive one has to be 'brave' and that seems to be the only thing eternal in my life! They say, I've committed a huge mistake, a sin perhaps, is it true though? If everything had a sleek and effortless answer like black and white, science would've never seen the light of day. The search of those countless inconspicuous colors amidst sleekness of striking black and white is itself what we call 'science'.

German bombers have burnt down a nearby town leaving no trace. But what seems uncanny is that plebeians aren't terrified of 'death', instead they just maledict the scourges.

If at all these maledictions served its purpose and turned into reality then war and savagery would have met an impeccable end! The world would be devoid of any suffering and agony.

My beloved Irene,my dear daughter, Here the earth never sees rain and is always wrapped with spine chilling snow!

# THE BEST OF The Tuesday toast

#### FEATURING WORKS OF

Agastya Pulapaka (II Maths) Deepanshu Bisht (II Physics) Rahul Mallikarjun (III Physics) Shyam Sunder (II Physics) Paul Joseph Robin (II Physics) Swapnila Chakrabarty (II Physics)

CURATED BY PAUL JOSEPH ROBIN



Brian A Skiff, an American astronomer noted for discovering numerous asteroids during his career, named three asteroids that he discovered in the 1980s after members of a world-famous quartet.

Interestingly, the last asteroid of this group carries the name that most would use as a connection between this quartet and the environment of an asteroid.

What did he name the asteroids after?

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This 17th century novel "The \_\_\_" was considered one of the first works of science fiction by the likes of Carl Sagan and Isaac Asimov. It describes in detail how earth might look like when viewed from the Moon. The author's name comes in the three famous scientific laws he formulated. The name of novel is something all of us have encountered and which relates to fiction or imagination.

When this element A was discovered, it used to be considered a high rank element. It was so popular that president of France B used to take his own dinner in plates of A while guests used to be served in gold or silver vessels. ID A & B.

Identify the author and novel.

ID X & Y.

A storm that lasted for months had put a strain on X which worked for 15 years against the intended few months as foreseen by the mission. Communications with X were basics numerical codes which were translated into information. The final communication from X were the numbers 22 and 10.8. 22 indicated a solar array energy production of 22 Wh for the sol, and 10.8, atmospheric opacity measured during the storm. The immensely popular quote Y which is a poetic translation rather than a literal one of X's last message is often cited as if a verbatim reproduction of X's final transmission.



#### THE TUESDAY TOAST

A B C D is a three-volume treatise, considered to be one of the most important works in the history of science. In the concluding essay to the second edition, the author used his famous expression, "I feign no hypotheses", in response to criticisms of the first edition.

C D is another three-volume work on the foundations of mathematics by famous philosophers. It is infamous for a proof of the validity of the proposition 1+1=2 spanning over a hundred pages. Nevertheless, the scholarly and philosophical interest is great: placing it 23rd in a list of top 100 English non-fictions. Name A B C D.

Compu- a computer brain with ten crore delicate circuits was invented by fictional character X along with several professors around the world and the Namura Institute of Japan. X happens to be one of the most famous comics created by Y. Y is the first Indian to receive an honorary Academy Award. ID X & Y



- 0 Beatles: Lennon, Harrison, Starr
  2 Opportunity, My battery is low, and it's getting dark
  4 Philosophiæ Naturalis Principia Mathematica
  - Answers:

- 5 Prof. Shonku, Satyajit Ray
  - III nosloqpN ,muinimulA č
- 1 Johannes Kepler, Somnium (The Dream)

# THE FIRST THREE VEARS

"Smart people learn from everything and everyone, average people from their own experiences, Stupid people already have all the answers"

- Socrates

Knowing more than people around me always made me feel good. It gave me a certain satisfaction. Although I would never attribute my choice of Physics to this fact, I have a sneaking suspicion that subconsciously, maybe this was indeed a major factor that led me to where I am today. An undergraduate studying Physics, a subject which has no dearth of knowledge.I had finally done it. I had joined the course of my dreams, where all the secrets of the universe would be unlocked and where I would achieve enlightenment in a short span of 3 years. I would be able to solve complex equations in my mind, map out intricate trajectories within seconds, be an expert in quantum entanglement and become the master of the cosmos (note the sarcasm). Alas, dreams are seldom true. By joining this programme, all my preconceived notions about Physics were shattered. My good friend from YouTube, Michio Kaku, was shunned for being a 'salesman for string theory', my ambitions for having more knowledge than people around me didn't go too well with many toppers from around the country in my class and my attempts at being a mindful citizen of planet Earth was rewarded with an 8 in Environmental Science! Physics was not what was shown to me in YouTube videos and popular science fiction movies. Physics is hard. I had been scammed. I thought that by joining a course in Physics, I would be free of all the rote learning. No. "Allow us to introduce ourselves", exclaimed Thermodynamics and Optics. Oh, very well. "I can handle rote learning", I told myself, "at least I am studying Physics. No one can take that away from me, right?". Wrong. Turns out you have to take a compulsory English (and EVS) course to "improve your vocuburly" (or something) and learn how to write a letter to some editor. Imagine writing a physical letter in the 21st century. "All right, I am not entirely studying Physics, but at least I am studying...". Negative. Ok that one is on me. And although sometimes I would wish that Thermodynamics should have never been a thing and that a building would have fallen instead of the Apple, I would only be frustrated with my lack of commitment. The joy of being in college was overshadowed by the daily 8 to 5 hectic routine and it always bugged me when someone from the arts department said they had classes till "late". On top of all that, one has to learn differently for the University exam?! For sure what is taught in the class must be sufficient to pass the exams with respectable marks. Nope. Never have I been so sad looking at my lucky number 7 than in my report card against Mechanics. Surely you are joking Mr. Report Card. GIVE ME A BREAK. "Damn it! I should have never taken this subject", a fresher in Physics might say after reading the above detailed roast of university Physics with only that as an account of their future. Some might even be tempted to quit while they are ahead.

I am sure millions of fresh undergraduates will read this article, but only to the trained eye will it be obvious that I have been telling you only half the story. Let us venture into part 2 of this article which I would like to call – 'The Second Part'.

One of the most important skills one can learn is to learn! There are a few better places you can **learn to learn** than by sitting and working through a Physics course in Stephen's. I would go so far as to say that I learned more about learning than about Physics. When I was in school, I really liked studying about different phenomena and concepts that were taught in class, but in college I understood what I was liking before were handwaving explanations without any deep understanding. Working through the first semester Mechanics course was a real-eye opener. It taught me how to think independently and learn by having an open mind, devoid of any prejudices. As a consequence of this, I always strive to understand the meaning of everything I study and not just memorise it. Although always comparing yourself to people in your class is not exactly a healthy habit. I think I made myself better by learning from people around me by

is not exactly a healthy habit, I think I made myself better by learning from people around me by inculcating their to the best of my ability. I have been fortunate enough to be a part of the wonderful amalgamation of people that is the 2018-21 Physics batch! I, as other people, would have liked to spend more time in college than the measly  $(\pi/2)e7$  seconds we got due to the pandemic but *it ees what it ees!* Much of the general population definitely got the shorter straw. One of the most exciting parts of studying Physics are the computational labs. I always look forward to undertake computational tasks since they teach so much about logic, equip us with serious marketable skills which can be used in many walks of life and more importantly, allow us to perform tasks which can't be done analytically. Along with these, we also have the experimental labs, which in my opinion, are a great source of practical knowledge given that many people dislike the idea of looking at a pendulum for 3 hours straight. They taught me a lot about taking good measurements, setting up better apparatuses (apparatii?) and messing with the readings so as to get better results. For legal reasons, that's a joke. When push comes to shove, the most important thing is self-study but it helps immensely if you have mentors who give the correct advice and the professors present in college, 100% fit this description. I have been very lucky to be a part of this community.

Funny that I wanted to learn about the universe (such a cliche) and ended up not taking the Astronomy course at all! This shows how interests and ideas can modify in such a short time scale. Whether you want to continue with Physics or not, the applications of the knowledge gained here manifest themselves almost everywhere, ranging from Economics to Biology to Computer Science to Sales to much more. I am currently going through a book on *Ikigai*, a Japanese word for 'a reason to jump out of bed in the morning'. Although I have not found my Ikigai yet, I believe my Brief History with Physics will help me do so, and in my very professional opinion, it will help you find yours too.

"We are what we repeatedly do. Excellence, then, is not an act, but a habit" ~Aristotle

RUDRA KALRA (3rd physics)



A true version of events.

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DOODLE CREDITS- SWAPNILA CHAKRABARTY

*"This damned war"*, said he. *"Now I can't go there too"*. The Physics Department at Allahabad was famed for its pioneering work. The research at that place on the atomic spectra and spectrometric analysis were known worldwide. Sommerfeld and Compton had both suggested it. He could also explore his lifelong interest in Vedanta, right at the place it started. He sighed and glanced at the letter again:

'His Excellency, The Viceroy, for the Emperor of India, deems it fit not to allow your visa for passage to India ..."

Then Schrödinger opened the next letter, from a good friend:

"... You are the only person who shares my view that reality is what can be experimentally verified. The Copenhagen spirit defies and violates all acceptable norms of reality. This spooky behaviour that particles remain in a mixed state and that no physical property can be understood without measuring one of the particles despite the separation between the two. As if the moon exists only when I look at it." He took the fountain pen and started writing back to Einstein:

'I quite agree with you. I don't like this; I hate that I ever had anything to do with it. The course it has taken from the foundation I built is just nonsense. In fact, I have stopped working on it. I have been reading your results on the Unified Field Theory and I must say I am impressed. I am working on it now and see some promising results.'

What he did not mention was that he had finally developed a simple solution to the problem. He was convinced his attempt at unifying the forces within Einstein's framework of relativity was quite successful. He would be presenting it at the Royal Irish Academy next week.

The lecture had gone well. The Prime Minister and many others had attended. He had finally found the solution to what had puzzled Einstein for years. The papers had picked it up too, *Schrödinger's Affine Field Theory presents a complete unified theory*'. Was his genius finally going to be acknowledged?

The next day headlines read, '*Einstein thinks Schrödinger's discovery just an attempt which is not complete*'. It further went on, '*The announcement is just preliminary. Since it is not backed by any physical evidence, the physics community fails to recognize it*'. He cursed out loud, "*That old git! He thinks himself the smartest of them all*".

Angered at this betrayal by the one person he trusted well, Schrödinger gave up the pursuit of his lifetime, physics. He spent most of his days visiting his colleagues.

"Figuring out how lifeforms evolve is more interesting than the secrets of the cosmos", he soon realized. There should be something like a complex molecule which contains a code for living beings. This code is stored in the molecules in some manner. "What is life?", he pondered, "How did we become who we are?" The rest of the time, he would keep writing. Consciousness, biology and even fiction.

And then one afternoon, at the home of one of his colleagues, Prof. March, an idea sparked. He was delighted. This would be his ultimate revenge. One from which they would never recover.He could finally prove that he was right, after all.

"Daddy, what is Madam March's cat doing here?", rushed in his thirteen-year-old, Eva.

"Nothing sweetheart, I just have an experiment to do", he explained. "I will put her into this box with a radioactive substance which will decay. Now, there's a counter which pops open a can of prussic acid when the first decayed atom is detected by it".

*"It'll kill the cat*", she cried.

"The thing is, there is as much chance that an atom will decay as it won't. And so unless you open the box, you can't know if the cat is alive or not". "But Daddy", said Eva, "won't the cat keep crying out, 'Meow' ?"

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# NOT ALL THOSE WHO WANDER ARE LOST

I think I can safely make the assumption that most people are no stranger to the title of this article. J.R.R Tolkien's famous words adorn one of the most intricate book series ever written, 'The Lord Of The Rings'. Well, the inspiration to write this article came impulsively whilst I was engrossed in a conversation with another friend regarding our approach to learning physics and how the rigorous curriculum of college life often limits us and our infinite fantasies.

I am no expert in this matter. What I am, is unabashedly romantic in my ways of looking at physics. What I write ahead is mere experience, a personal tale I wanted to share and I hope no one takes offence.

I was and hopefully, still am one of those starry-eyed kids, with immense faith in the power and beauty of science. I don't think my ambition for physics came from the fact that I wanted to learn everything about the universe (not gonna lie, I still kinda wanna work for ISRO) but from a simplistic desire to learn, learn whatever comes my way and learn it well. I realise that whatever I said just now makes me sound like some kind of learned philosopher but I, with the power of my self-acclaimed stupidity can assure you that I am not. Getting back to point, I belong to the class of people who like knowing each and every detail of something they are introduced to. As a high school kid, I shunned the rigidity of a syllabus and read each page, each line and each word of every book that was handed over to me. Guess that's why I didn't do very well in one of those exams where you go to coaching centres to learn elaborate tricks to solve questions you don't know head or tail of.

"This is how you solve it if it comes in the exam."

"Why? Why is it solved this way?"

"You just need to know it is solved this way."

Therefore after this holy grail of high school experiences, I decided that physics is my ultimate gateway to learn everything in depth and also to rebel against the very Indian notion where students must take up engineering or medical studies as soon as they have a socially validated 'good brain'. I was satisfied with myself when I joined Stephen's and I assured myself that it is going to work out. I will not say it didn't but not in the way I thought it would. The best part of being in this college is that you are actually encouraged to learn and explore and that is something priceless but college life is seldom restricted to that. You also join something that I like to call 'The League of the Nerds' because that is exactly what happens when everyone in your class has the history of being a topper. Now, marks have always been secondary for me but when I see people crying over one or two marks while I am overjoyed about solving that one problem correctly, the one I worked on the entire night before the exam, I am bound to feel weird and incompetent about myself. At some point studious regular classmates encourage you to put in more effort into what you do but at some point it also makes you look at all of it as some rigorous formality. Now, going to weddings is something you do out of formality but studying is absolutely not that, at least for me. I completely understand that as a student it is our duty to be regular and I also acknowledge my failure at it for some part of my life but I absolutely hate it when a sense of competition is attached to everything you do. And all this stress of involuntarily being in a constant battle to be better takes a toll on people who look at their subject with a sense of passion rather than duty.

Honestly, I have been hard on myself for the first three semesters of my college life. I have been hard on myself because I wasn't good at mechanics despite trying . I have been hard on myself because I was battling with depression and hated the fact that my grades were suffering. I have been hard on myself because I couldn't be as good as I wanted to be. I have been hard on myself because I was losing all my adoration for a subject I loved the most. The truth is, people hardly dare to speak up about the downsides of something that has been silver-coated in their minds. So, in simple words, life here is a race and not many really want to slow down for the more humane things. Slowing down for some heart-to-heart talks, slowing down for a fun day around the campus, slowing down for a week dedicated to binge-watching a science sitcom or reading a big fat book completely out of syllabus or even slowing down for an extra cup of coffee. The irony being that we are actually encouraged to slow down and enjoy the process but almost everyone is so engrossed in keeping up with the expectations and tags of being here, they forget that there is so much more to life. I am sounding so very controversial right now. \*makes gasping noises\*

But I eventually met and became friends with romantics like me. People with an unquenchable thirst for knowledge and odd tendency of rebellion. Believe me, we have no dearth of panic in our lives. We go into libraries and panic over the all the books we are not reading, we panic before exams that we haven't studied all the recommended books in the syllabus, we panic over not knowing every intricate aspect of a lab setup, we panic over not visiting every citation in a research paper, we panic over the not being able to watch the remaining episodes of a science mystery show, basically we panic over everything we don't know and it is not limited to physics.

So essentially, the term for us is 'wanderers'. We are lost in the vastness of the ocean of knowledge and we are not sure which shore to swim up to. We are crippled with the inability to choose. I wonder if I am really in love with the subject, how on earth am I supposed to choose between the dance of the planets and the ka-boom of the atoms. \*sounds so magnificently childish\* But that's us, we don't like a laid out rigid path, we like a little bit of everything in life ranging from rock music to k-pop, nuclear machinery to robotics, anime to british sitcoms. And looking at people who have decided what to do with their lives, we only feel a little demotivated and suffocated by the necessity of hardcore choices. On some days we want to write research papers all day long and on some days we want to be Christopher Nolan's side-kick. I understand that our indecisiveness comes as a result of youth and our tendency to wander but I think uncertainty has a beauty of its own. That's why quantum is such a loved field, isn't it? \*we never know\* The thing is being a wanderer doesn't imply we are on the path of least resistance. It involves the most pain as eventually we will need to boil down our choices, rationalize them and have a sense of dissatisfaction permeate our lives but I think that's a risk that can be willingly taken. To always feel like we don't know enough only encourages us to know more. That's the beauty of physics anyway, it condenses the universe, every aspect of it into concepts and theories and while making our way through it we get a taste of almost everything life can throw at us.So being a wanderer in physics will always be one of the best things to have happened to me. I will not judge myself and advise you to not judge yourself by grades or internships or classmates who are better than you(but also remember those things are important and you should put in the effort). If you are a wanderer like me, physics will eventually teach you to claim what's yours and you will bounce back. That is exactly what Tolkien wanted us to know that it is okay to wander as long as you do not lose the desire to make your way back. And as Paulo Coelho puts it, 'And, when you want something, all the universe conspires in helping you to achieve it'.

So, I have been lost, bewildered, sad, guilty, regretful and gloomy. At one point point I have felt so very incomplete and had hated myself for losing interest in physics but every time I concluded that what physics births in me is a desire of knowing little things from all around and at some point I will find something fulfilling enough to dedicate my entire life to, which doesn't necessarily has to be physics itself. Physics for me, is hope and hope is a good thing, probably the best of things and no good thing ever dies.

#### HERE'S QUOTING A BELOVED SONG OF MINE, JUST TO IMPLY THAT IN HERE, YOU WILL ALWAYS FALL BACK WHERE YOU BELONG:

"I still feel alive When it is hopeless, I start to notice And I still feel alive Falling forward, back into orbit So, when I lose my gravity in this sleepy womb Drifting as I dream, I'll wake up soon To realize the hand of life is reaching out To rid me of my pride, I call allegiance to myself"

Physics is not just my undergraduate degree, it's an experience, the college, the professors and the people. It is quite saddening that the lockdown deprived so much of an authentic college feeling from all of us. The labs, the classroom chatter, the debates and passionate discussions, everything about it. Online education feels like a red tape procedure, it has no real essence. But I guess we adapt, we thrive in our difficulties. Lastly, physics is a storm I voluntarily walked into, a storm I don't regret.

"And once the storm is over, you won't remember how you made it through, how you managed to survive. You won't even be sure, whether the storm is really over. But one thing is certain. When you come out of the storm, you won't be the same person who walked in. That's what this storm's all about" - Haruki Murakami

#### SWAPNILA, SECOND PHYSICS

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# ALIENS

#### A story by Adis

It was a quiet night.

He had had enough of school. They barely understood his potential, his calibre. They needed to start taking him seriously. Those tests? They didn't matter, what would he do with literature or geography? His future was up there.

He looked up at the sky. So many stars, so many planets, maybe one of them had life? He wondered everyday. It was impossible, that with so many stars just in the known universe, that none of them held life. There had to be aliens somewhere.

He decided to wander into the forest. He wasn't scared of the dark or the wildlings. They knew him well enough and by now, they considered him one of them. He had nothing to f ear.

His mind lost in thoughts of space exploration, and the hopes of one day finding an alien he made his way to the stream.



It was a small brook, more than a stream, just a small trickle of water, but the sound it made as it flowed over the rocks calmed him. He couldn't feel anger at this spot, not even at his maths teacher.

#### CRACK.

He jumped.

He looked around. Nothing. Probably an animal.

He shrugged and looked back at the sky, losing himself in the stars.

#### CRACK.

This one was closer. He decided to check, what if an animal was trapped in a trap or something?

He got up and jumped across the stream, making his way to where he felt the noise had come from.

As he made his way into the darkness of the trees, he felt he could see a faint light coming from somewhere in the woods, from somewhere a light had no business being.

As he got closer to the light though, he stopped. Four bipedal beings, shiny, and sparkling in the moonlight. Their faces were round and bulbous, and seemed to reflect the light around them. He stood there, transfixed. He had seen the aliens.

But the aliens had seen him too.

With guttural, piercing yells, they leapt towards him, and before he could run, they had him bound. They shook him, and said something in their raspy tongue, which he couldn't understand. He tried to shake the captors away but they were too strong. Their leader, or so it seemed to the boy, closed his shiny hand and made a fist. And the boy's captor began to drag him towards the leader. The leader began to walk deeper into the woods with the boy being dragged behind him.

**Y**1

After what felt like an eternity in the woods, they reached a clearing, and in the clearing was the most wondrous thing the boy had ever seen. A large, metallic structure, the kind the boy had never seen. This was probably the aliens' spaceship, and this theory was confirmed when with a hiss and a release of smoke, a stairway descended from the structure and the aliens pushed him into it.

The boy had given up his struggle by then. This was his destiny. He knew he had to go through with this and learn more about these wondrous beings. Besides, if he behaved nice to them, maybe they would make him their ambassador!

The aliens latched the boy into a seat, latched themselves up as well, and throttled the aircraft. Within minutes, the craft was airborne, and to the boy's wonder, the craft headed towards an even bigger craft hovering over the planet's atmosphere, and slowly latched on to it. The boy began to try to make conversation, he couldn't

resist anymore.

"Who are you?"

The alien holding him made the raspy sound yet again and shook him. The boy took that as his clue to shut up.

With a great hiss of smoke, the doorway of the craft opened, and the alien holding him hauled him out onto the bigger craft.

On the bigger craft though, the aliens looked nothing like the ones that had caught him.

For one, they weren't shiny, they were many different colours, and all of them had two layers of skin, one that looked like normal skin, and another that looked like fabric, that draped over their normal skin. This confused the boy, why would ANYONE need two skins?

They didn't have bulbous, reflective heads either. Their heads were round, or triangular, or somewhere in between, and a few of them had weird keratinous growths on their face, while almost all of them had this keratinous growth on their head or their bodies.

The boy turned to look at his captors, and was astounded to find that their bulbous heads were now in their hands, and they were in the process of removing their shiny skins to reveal fabric skins. These aliens could discard skins. Oh, the marvels.

One of the younger looking aliens rushed over to him, and

started poking him. This alien had one of the longest keratinous growths in the place, which was a beautiful shade of gold. The boy was transfixed, and suddenly without warning, he could understand all the rasping around him, which he had almost got used to.

"..... turn on the translator!"

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"Well, we need to tell him and he needs to understand what we are going to do!" "Why must he? He is never going to go back to his planet anyways!!! We have no place for sentiment Sergeant! You know what we are here for!"

But he's just a child, almost a baby!"

"Well, that's the best we could get! We will live with this.

Let's go back now, we've spent long enough in space."

"But ...."

"No buts. Let's go. Prep the engines."

The boy couldn't understand a lot of it, except the part where they said he couldn't ever come back. He was scared, but also thrilled, were they taking him to their home planet?

But then, the alien with the golden hair grabbed him.

"Listen, if I get a chance, I'll help you escape. They are going to experiment on you, they are going to use you for their scientific advancement, but you're just a baby. I can't let that happen..."

#### BZZZZZ

"SERIOUSLY SERGEANT? TREASON? YOU'RE GONNA BE IN HYPERSPACE FOR THE REST OF THE TRIP, AND I'LL DECIDE WHAT TO DO WITH YOU WHEN WE ARE BACK!" The leader had pressed a long metallic tube onto the alien's neck, and it had fainted.

The boy shuddered. These weren't the aliens he had hoped for. They were cruel, dangerous and didn't mind hurting their own.

"Sir! The engines are ready!"

"Good, engage the FTL drive."

"On it, sir."

The boy felt a dread he had never felt before. But he continued to listen to the aliens speak.

"It's a good thing we got him he would be able to tell us a lot about their defences."

"Him? He's just a kid. Why would he know anything?"

"Well, anything he tells us, is a lot more than they will know about us, when we attack"

"So, the invasion has been finalised?"

"Well yes. You know our situation we need more space and resources for everyone. This planet is perfect. It's almost exactly like ours. "

"That it is, wonder why the aliens are so different from us though?"

"Sir, the FTL drive is online!"

"Good. Set course for Earth. Let's go back home. "

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The First Years made it to the college in circumstances like no other! With a chunk of their board exams cancelled, disengaging apprehension looming over the competitive exams and their last few moments of high school snatched away, they had it really tough. With the semester delayed, they have a very heavily packed up first year ahead to tackle and conquer.

One of their own, Shiny Sharon J. shares her experience of making it to Stephen's.

"It's has been the most bizarre year in the last few decades. In the midst of this haunting pandemic, the schools bid farewell to its students in the most unusual way. All thanks to the virus which delayed its launch in the country, thereby letting us have the bare minimum qualification in our academics.

In those times of absolute uncertainty, with so many questions arising, it was fully upon me to make my next move. The interests I possessed acted as a function and the bits of liking that I had for the subject (which is known for laying down the laws of nature) played the rule of input. Yes, I started looking for an amazing platform to go on with this passion of mine. Ah! the way my imaginations grew. Sitting in a lecture hall with other pupils and witness mind-blowing lectures and splendid experiments and try and understanding the beauty of it. But at the same time, I knew that in reality, my learning experience is going to be completely different.

#### After

much time spent being a thoughtful waste-woman, I did something productive

rather than just imagining. Ended up applying for this course, in this place where I believed that I can expand my zone of exploration. And soon enough the dreaded

interview day came along. The chillness in the weather around me helped me maintain my equilibrium and I feverishly prayed that nature be with me. The interview which lasted for 1/48 of the time period of earth's rotation for my interviewers, spanned much lesser than that in my frame of reference. After facing the scholars, I had made up my mind, my probability of getting selected is same as that of occurrence of rainbows on mercury. Good Lord! But I made it and now I am going crazy studying everything I wished to. So yes, nothing was planned, it all just unraveled. "

## "what made you land here?"

A GOOGLE FORM WAS SENT OUT TO THE FIRST YEARS

### AND THEY HAD SOME AMUSING ANSWERS

The engineering thing didn't work out, but with me clearing the SSC interview, fate decided that I shall become a physicist (optimistic me selfproclaiming).

I really don't know why exactly I chose physics. I was good at math but hated doing just math. I do give a lot of credit to me reading about Richard Feynman. As a physics major we are compelled to complain about how hard physics is but really cant imagine myself studying anything else:)



Earlier I thought to go for mechanical engineering is just because I had changed the plug for switch board when I was in 8th. physics is just a medium for me to be professor and I don't like it in depth and that's why don't want to become physicist. But isn't who is studying physics, a physicist already!!!!

 $\bigcirc$ 

OFF

ON

 $\bigcirc$ 

To observe, to practise and to conquer...

A seed of curiosity was planted in me by a teacher of mine. I guess, the notion of physics being a subject of concepts and ideas derived from reading the story of nature itself has tickled my brain.

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Just the sheer lack of knowledge of humankind about nature drives me closer to physics

It sounds quite cool when I hear about CERN and TIFR and such organizations and I think that studying physics might give me an opportunity to gain skill sets to work with these organizations.

### 'Applied Maths and Carl Segan'

 $\bigcirc$ 

OFF

 $\bigcirc$ 

I got both physics and chemistry. I like both and I was really confused. So, I called both my teachers. Chemistry ma'am didn't pick up my call. Physics ma'am did. So, I took physics.

## 'To be confused'

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## **ASTROPHOTOGRAPHY** BY VEDANTA THAPAR

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This image of the moon, taken from a 200mm Reflecting Telescope, represents its waxing crescent phase. 2 of the most prominent darker hue spots on this image are the Sea of Serenity(Mare Serentitatis) and the Sea of Tranquility (Mare Tranquillitatis). The Sea of Tranquility is of great significance as it hosts the crater that was the landing site of the first manned mission to the moon, Apollo 11, in 1969.



This image, taken from a 60mm Refractor and a Phone, represents the famous Great Conjunction of Saturn and Jupiter. This image was actually taken after a 50x zoom and it beautifully shows how close Saturn and Jupiter actually were even in the eyepiece. Infact to the naked eye they appeared as one slightly elongated star. You can see Jupiter with 4 of its giant moons along with Saturn to its right. This conjunction coincided with the winter solstice on 21st December 2020 and was visible to us for the first time in about 800 years

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This image is a long exposure of the winter night sky taken close to Midnight on a moonless night. It is a 30s exposure taken from a 64MP Camera of a mobile phone. One can see the beautiful hues of the Milky Way and make out some of the dust clouds towards the centre of the galaxy This image, taken from a 60mm Refractor and a Phone, shows a 30s double exposure of NGC 1977: a blue reflection Nebula next to the Orion Nebula. The Orion Nebula lies inside the Orion Constellation(known as the Hunter) which is one of the most noticeable and beautiful constellations in the winter sky. One can view the hubble picture of this Nebula online to see even more details of its structure



## **PHYSICS SOCIETY**

2021


## Stage 1: Starry eyed

The world is my oyster. I will solve all the mysteries of the universe with my knowledge in quantum and astro, Ah physics!! Absolutely beautiful.

## Stage 2: Hanging in there

I have no social life or sleep but just one more lab assignment! quantum and astro are waiting for me, I will get to it, I have to.

## Stage 3: The Third year



DOODLE CREDITS: SANDRA E. S. DOODLE CONCEPT: SWAPNILA

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