Name of Course	: **CBCS B.Sc. (Physical Sciences)-II**
Unique Paper Code	: **42354302_OC**
Name of Paper	: **Algebra**
Semester	: **III**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find the inverse of the element $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ in $GL(2, \mathbb{Z}_7)$.

Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in R, a \neq 0 \right\}$. Show that G is a group under matrix multiplication.

2. Prove that a group G is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a and b in G.

How many subgroups are there, of cyclic group $\mathbb{Z}_{40}$. Find them and write the elements of all the subgroups.

3. Let $= \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$. Compute each of the following

(i) α^{-1}, (ii) $\beta\alpha$, (iii) $\alpha\beta$.

Also find the orders of $\alpha, \beta, \alpha^{-1}, \beta\alpha, \alpha\beta$.

4. Show that the set $S = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} : a, b \in \mathbb{Z} \right\}$ is a subring of the ring M_2 of 2×2 matrices over integers.

Prove that the only ideals of a field F are $\{0\}$ and F itself.

5. Let $V = \{a_2x^2 + a_1x + a_0 : a_0, a_1, a_2 \in \mathbb{R}\}$ be a vector space over $\mathbb{R}$. Show that $\{x^2 + x + 1, x + 5, 3\}$ is a basis of V.

Let $V = \mathbb{R}^3$ and $W = \{(a, b, c) : a, b, c \in \mathbb{R}$ and $a + b = c\}$. Is W a subspace of V? If so, what is its dimension?

6. Which of the following maps $T: \mathbb{R}^2 \to \mathbb{R}^2$ are linear transformations?

(i) $T(x_1, x_2) = (1 + x_1, x_2)$, (ii) $T(x_1, x_2) = (x_2 - x_1, 0)$.

Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $T(2,1,0) = (1,2)$, $T(1,0,-1) = (-1,1)$ and $T(0,3,1) = (1,3)$. Find $T(5,3,-1)$.