Name of Course	: **CBCS (LOCF) B.Sc. (H) Mathematics**
Unique Paper Code	: **32351101**
Name of Paper	: **BMATH 101- Calculus**
Semester	: **I**
Duration	: **3 hours**
Maximum Marks	: **75**

Attempt any four questions. All questions carry equal marks.

1. Sketch the graph of the function $f(x) = x^3 - 6x + \frac{5}{2}$ by finding intervals of increase and decrease, relative extrema, concavity and inflection points (if any).

Evaluate the limit: $\lim_{x \to -\infty} (1 - \frac{1}{2x})^{5x}$.

A shoe factory can produce shoes at a cost of \$70 a pair and estimates that if they are sold for x dollars a pair, customers will buy approximately $N(x)$ number of pairs per week, where $N(x) = 500\, e^{-0.2x}$. Find the price at which the shoe pairs be sold so as to maximize the gain.

2. Find all horizontal and vertical asymptotes to the curve $y = \frac{x^3+8}{x^3-8}$. Does this curve have any cusp or vertical tangent? Explain.

Find the n^{th} derivative of the function $y = 3\sin^3 x$.

Obtain a reduction formula for $\int \operatorname{cosec}^n x\, dx$ and hence evaluate $\int \operatorname{cosec}^7 x\, dx$.

3. Sketch the graph of the curve $r = 1 - 2\sin\theta$ in polar coordinates.

Describe the graph of the equation: $5x^2 + 9y^2 - 20x + 54y + 56 = 0$.

Identify and sketch the curve: $3x^2 + 2\sqrt{3}\, xy + y^2 - 8x + 8\sqrt{3}\, y = 0$.

4. Find the arc length of the parametric curve: $x = (1 + t^2),\ y = (1 + t^3)$ for $0 \le t \le 1$.

Find the area of the surface generated by revolving the curve $y = \sqrt{25 - x^2}\,,\ -2 \le x \le 2$ about x-axis.

The region bounded by the curves $y = x$ and $y = x^2$ is rotated about the line $y = 4$. Compute the volume of the resulting solid.

5. Given $9\sinh x - \cosh x = 5$. Find the exact value of $\tanh x$.

Show that the vector- valued function given by

$$\boldsymbol{R}(t) = (2\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}}) + \left(\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} - \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}}\right)\cos t + \left(\frac{1}{\sqrt{3}}\hat{\boldsymbol{\imath}} + \frac{1}{\sqrt{3}}\hat{\boldsymbol{\jmath}} + \frac{1}{\sqrt{3}}\hat{\boldsymbol{k}}\right)\sin t$$

describes the motion of a particle moving in the circle of radius 1, centered at the point $(2, 2, 1)$ and lying in the plane $x + y - 2z = 2$.

A ball is thrown upward from the edge of cliff at an angle 30^o with initial speed 68 ft./s. Suppose the height of the cliff from the ground is 50 ft., then find the velocity and speed of the ball at the time of impact. Also, find the highest point where, the ball reached during the flight.

6. Find the integral $\int_1^e \frac{dx}{x\sqrt{1+(\ln x)^2}}$.

A shell is fired from the ground level with muzzle speed of 450 ft./s at an angle of 30°. An enemy gun 23,000 ft. away fires a shot 2 seconds later and the shells collided 45 ft. above the ground at the same speed. What are the muzzle speed V_o and angle of elevation α of the enemy gun?

Suppose $\boldsymbol{R}(t) = t\hat{\boldsymbol{\imath}} + 2t^2\hat{\boldsymbol{\jmath}} + t^3\widehat{\boldsymbol{k}},\ t \geq 0$ is the position vector of a moving object. Find the tangential and normal component of objects acceleration.