Name of the Department	:	Physics
Name of the Course	:	B. Sc. (H) Physics – CBCS - NC (LOCF)
Semester	:	I
Name of the Paper	:	Mathematical Physics I
Unique Paper Code	:	32221101
Question Paper Set Number	:	B
Maximum Marks	:	75

Time Duration: 3 hours
Instruction for Candidates

1. Attempt **FOUR** questions in all.
2. All questions carry equal marks.

1. Solve the following first order differential equations

a. $\left(x+y+1\right)\frac{dy}{dx}=1$

b. $y\,dx+\left(2x-y\,e^{y}\right)dy=0$

c. A metal bar is kept at a temperature of $100^{o}F$. It is placed in a room whose temperature is $10^{o}F$. After 20 minutes, the temperature of the metal bar is $55^{o}F$. How much time will it take for the metal bar to reach a temperature of $20^{o}F$? Determine the temperature of the metal bar after 10 minutes.

2. Solve the following second order differential equations

a. $\frac{d^{2}y}{dx^{2}}+9y=sec\,3x$

b. $\frac{d^{2}y}{dx^{2}}-4y=x^{2}e^{3x}$

c. $\frac{d^{2}y}{dx^{2}}-3\frac{dy}{dx}+2y=\left(1+e^{-x}\right)^{-1}$ (Use the method of variation of parameters)

3. Find the shortest distance from the point $\left(1,-2,1\right)$ to the plane determined by three points $\left(2,4,1\right),\left(-1,0,1\right)$ and $\left(-1,4,2\right)$.

Find the value of $'a'$ for which the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+a\hat{j}+5\hat{k}$ are co-planar.

If $f\left(r\right)$ is differentiable, then show that $f\left(r\right)\vec{r}$ is irrotational.

4. If ϕ is a scalar field and $\vec{A}$ is a vector field, then prove that, $\vec{\nabla} \circ \left(\phi \vec{A}\right) = \left(\vec{\nabla}\phi\right) \circ \vec{A} + \phi\left(\vec{\nabla} \circ \vec{A}\right)$. Hence evaluate, $\vec{\nabla} \circ \left(r^3 \vec{r}\right)$

Show that, $\iiint_V \vec{\nabla}\phi dV = \iint_S \phi \hat{n} dS$

5. Evaluate $\int_C \vec{F} \circ d\vec{r}$ where $\vec{F} = \left(x - 3y\right)\hat{i} + \left(y - 2x\right)\hat{j}$ and C is the closed curve in the xy-plane, $x = 2\cos t, y = 3\sin t$ from $t = 0$ to $t = 2\pi$.

Verify the divergence theorem for $\vec{A} = 2x^2 y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$ taken over the region in the first octant bounded by $x^2 + z^2 = 9, x = 2$.

6. Obtain the expression for curl of a vector in orthogonal curvilinear coordinates and express it in spherical coordinates. In spherical coordinates, verify, $\vec{\nabla}\frac{1}{r} = \vec{\nabla} \times \left(\cos\theta \vec{\nabla}\phi\right)$

In a football match, an average of 3 goals are scored in each game. What is the probability that 4 goals are scored in a game?