1. Apply Dijkstra’s Algorithm OR Improved version of Dijkstra’s Algorithm to find a shortest path from A to F, also write steps wherever possible.

In the pseudograph given below either describe an Eulerian circuit or explain why no Eulerian circuit exists.

2. Prove or disprove the statement: Homomorphic image of modular lattice is modular.

Construct a lattice $L$ with 0 and 1, so that $L$ has at least one element having three complements.
Verify whether the lattice given below is modular and /or distributive, by using M3-N5 theorem.

Find the disjunctive normal form of the Boolean polynomial \( p = (xy' + xz)' + x' \). Further, find the conjunctive normal form of 'p'.

3. Explain the Königsberg bridge problem and discuss the solution provided by graph theory to this problem. The degree of each vertex of a certain graph is either 4 or 6. The graph has 12 vertices and 31 edges. How many vertices of degree 4 are there? Draw the subgraphs \( G \setminus \{e\} \), \( G \setminus \{v\} \) and \( G \setminus \{u\} \) of the following graph \( G \).

Find the adjacency matrices \( A_1 \) and \( A_2 \) of the graphs \( G_1 \) and \( G_2 \) shown below. Find a permutation matrix \( P \) such that \( A_2 = PA_1P^T \).

4. Is the expression \( y'z' \) an implicant of the expression \( xy'z' + x'y + x'y'z' + x'yz \). Give reasons for your answer.
What are prime implicants of \( p = xyz + xyz' + xy'z + x'yz + x'y'z \)?

Using K-maps or Quine–McCluskey method, find the minimal sum of products form of the polynomial \( p \).

Give the symbolic representation of the circuit \( q = (x'yz)' + x'y'z' + (xy'z)' + xy'z' \).

Also, draw the contact diagram of above circuit \( q \).

5. Let \( X = \{1, 2, 3\} \). Consider the partial ordered set \( (L, \leq) \) where \( L = P(X) \) is the power set of \( X \) and \( \leq \) is defined as, \( U \leq V \) if and only if \( U \subseteq V \ \forall U, V \in L \). Draw Hasse diagram of \( (L, \leq) \). Prove or disprove that \( (L, \leq) \) is a chain. Justify your answer. Find a subset of \( (L, \leq) \) that forms a chain with respect to the same partial order relation.

Consider poset \( Q = \{a, b\} \) where \( a < b \). Is the map \( \theta : L \to Q \) order preserving where

\[
\theta(U) = \begin{cases} 
  a, & \text{if } U = X \\
  b, & \text{if } U \neq X 
\end{cases}
\]

Justify your answer.

Exhibit an order isomorphism between the given partial ordered set \( L = P(X) \) and partial ordered set \( S \) of all positive divisors of 30, with respect to the order that for any \( a, b \in S \), \( a \leq b \) if and only if \( a \) divides \( b \). Are the Hasse diagrams of two partial ordered sets \( (P(X), \subseteq) \) and \( (S, \leq) \) identical?

State a result describing a relationship between the existence of an order isomorphic map between any two finite ordered sets \( A \) and \( B \) and their Hasse Diagrams. Can you prove this statement?

6. Let \( L_1 = \{2, 4, 8, 10, 20, 40\} \) and \( L_2 = \{1, 2, 4, 5, 20\} \) be partially ordered sets with divisibility as the partial order relation. Are \( L_1 \) and \( L_2 \) lattices? Justify your answer. Show that the collection of all subgroups of a group \( G \) forms a lattice.

Consider lattices \( L_3 \) and \( L_4 \) represented by the Hasse diagrams shown below

\[L_3\quad L_4\]

Draw the Hasse diagram of lattice \( L_3 \times L_4 \).

Give example of a subset \( S \) of a lattice \( L \), which is not a sublattice of \( L \) but is itself is a lattice.