Name of the Course : **B.Sc. (H) Mathematics**

Unique Paper Code : **32357505**

Name of the Paper : **DSE-II Discrete Mathematics**

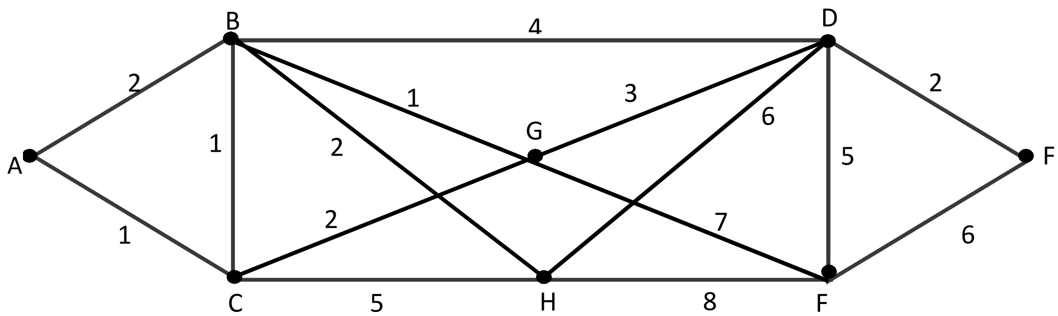
Semester : **V Semester**

Duration : **3 hours**

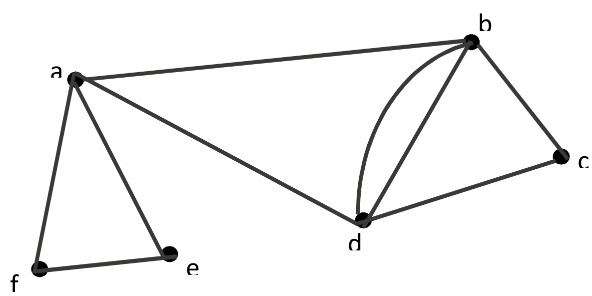
Maximum Marks : **75 Marks**

*Attempt any four questions. All questions carry equal marks.*

**1.** Apply Dijkstra's Algorithm OR Improved version of Dijkstra's Algorithm to find a shortest path from A to F, also write steps wherever possible.

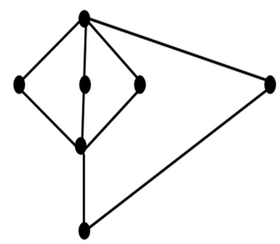



In the pseudograph given below either describe an Eulerian circuit or explain why no Eulerian circuit exists.




**2**. Prove or disprove the statement: *Homomorphic image of modular lattice is modular.*

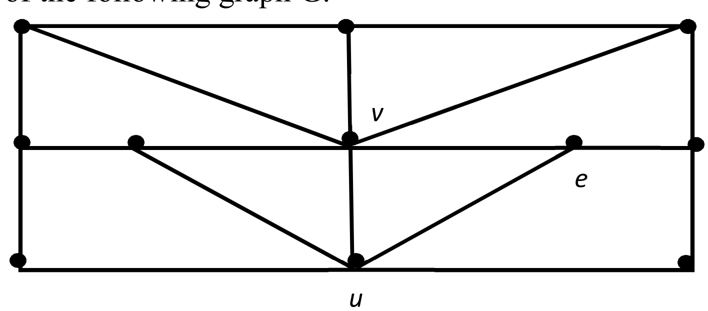
Construct a lattice $L$ with 0 and 1, so that $L$ has at least one element having three complements.



Verify whether the lattice given below is modular and /or distributive, by using M3-N5 theorem.

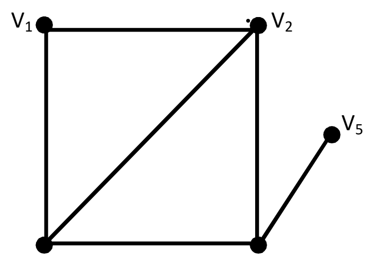
Find the disjunctive normal form of the Boolean polynomial $p = (xy' + xz)' + x'$. Further, find the conjunctive normal form of '$p$'.

**3**. Explain the Königsberg bridge problem and discuss the solution provided by graph theory to this problem. The degree of each vertex of a certain graph is either 4 or 6. The graph has 12 vertices and 31 edges. How many vertices of degree 4 are there? Draw the subgraphs G $\setminus\{e\}$, G $\setminus\{v\}$ and G $\setminus\{u\}$ of the following graph G.

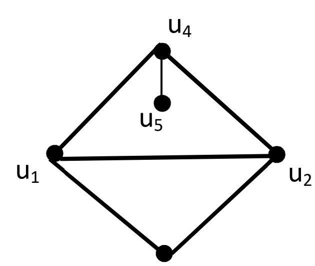



**G**

Find the adjacency matrices $A_1$ and $A_2$ of the graphs $G_1$ and $G_2$ shown below. Find a permutation matrix $P$ such that $A_2 = PA_1P^T$.




$\mathbf{G_1}$




$\mathbf{G_2}$

**4**. Is the expression $y'z'$ an implicant of the expression $xy'z' + x'y + x'y'z' + x'yz$. Give reasons for your answer.

What are prime implicants of $p = xyz + xyz' + xy'z + x'yz + x'y'z$ ?

Using K-maps or Quine–McCluskey method, find the minimal sum of products form of the polynomial $p$.

Give the symbolic representation of the circuit $q = (x'yz)' + x'yz' + (xy'z)' + xy'z'$.

Also, draw the contact diagram of above circuit $q$.

**5.** Let $X = \{1, 2, 3\}$. Consider the partial ordered set $(L, \leq)$ where $L = P(X)$ is the power set of $X$ and '$\leq$' is defined as, $U \leq V$ if and only if $U \subseteq V \quad \forall U, V \in L$. Draw Hasse diagram of $(L, \leq)$. Prove or disprove that $(L, \leq)$ is a chain. Justify your answer. Find a subset of $(L, \leq)$ that forms a chain with respect to the same partial order relation.

Consider poset $Q = \{a, b\}$ where $a < b$. Is the map $\theta: L \to Q$ order preserving where

$$\theta(U) = \begin{cases} a, & \text{if } U = X \\ b, & \text{if } U \neq X \end{cases}.$$

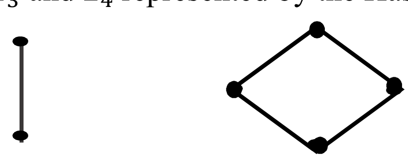
Justify your answer.

Exhibit an order isomorphism between the given partial ordered set $L = P(X)$ and partial ordered set $S$ of all positive divisors of 30, with respect to the order that for any $a, b \in S$, $a \leq b$ if and only if $a$ divides $b$. Are the Hasse diagrams of two partial ordered sets $(P(X), \subseteq)$ and $(S, \leq)$ identical?

State a result describing a relationship between the existence of an order isomorphic map between any two finite ordered sets $A$ and $B$ and their Hasse Diagrams. Can you prove this statement?

**6.** Let $L_1 = \{2, 4, 8, 10, 20, 40\}$ and $L_2 = \{1, 2, 4, 5, 20\}$ be partially ordered sets with divisibility as the partial order relation. Are $L_1$ and $L_2$ lattices? Justify your answer. Show that the collection of all subgroups of a group $G$ forms a lattice.

Consider lattices $L_3$ and $L_4$ represented by the Hasse diagrams shown below



$L_3$ $L_4$

Draw the Hasse diagram of lattice $L_3 \times L_4$.

Give example of a subset $S$ of a lattice $L$, which is not a sublattice of $L$ but is itself is a lattice.