Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357501**
Name of Paper	: **DSE-I NUMERICAL METHODS**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Consider the equation $2x - \log_{10} x - 7 = 0$ on $]3.78, 3.79[$. Find n, the number of iterations of bisection required to have an approximate root with absolute error less than or equal to 10^{-7}.

Find an approximate root for the equation $f(x) = x\,\sin x - 1 = 0$ using Regula-Falsi method. Do 2 iterations.

Use secant method to find an approximate root of the equation $f(x) = x^2 - 2x + 1 = 0$ starting with $x_0 = 2.6$ and $x_1 = 2.5$. Do two iterations. Compare the results with the exact root $1 + \sqrt{2}$.

2. Find the maximum value of the step size h that can be used in the tabulation of $f(x) = \sin 2x$ on $[1,2]$ so that the error in the linear interpolation of $f(x)$ is less than 5×10^{-4}.

The function $f(x) = x^3 + 2x^2 - 3x - 1$ has a zero on the interval $[-3, -2]$ and another on the interval $[-1, 0]$. Approximate the largest negative zero using bisection method. Do 2 iterations.

The equation $x^2 + ax + b = 0$ has two real roots α and β. Consider a rearrangement of this equation as $x = -\dfrac{(ax+b)}{x}$. Show that the iteration $x_{i+1} = -\dfrac{(ax_i + b)}{x_i}$ will convergent be near $x = \alpha$, when $|\alpha| > |\beta|$.

3. Find an LU decomposition of the matrix $A = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 3 & 8 & 2 \end{bmatrix}$ and use it to solve the system $AX = [1, 2, 8]^T$.

For Jacobi method, calculate T_{jac} and c_{jac} and the spectral radius of the coefficient matrix of the following system

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} X = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

4. Solve the following system of equations using Gauss Seidel iteration method:

$$2x_1 - x_2 + x_3 = -1$$
$$-x_1 + 4x_2 + 2x_3 = 3$$
$$x_1 + 2x_2 + 6x_3 = 5$$

Take $X^{(0)} = [0, 0, 0]^T$ and iterate three times.

Solve the following system of equations using SOR iteration method:

$$2x_1 - x_2 + x_3 = -1$$
$$-x_1 + 4x_2 + 2x_3 = 3$$
$$x_2 + 6x_3 = 5$$

Take $w = 0.9$ with $X^{(0)} = [0, 0, 0]^T$ and iterate three times.

5. Approximate the derivative of $f(x) = \sin 2x$ at $x_0 = \pi/2$, taking $h = 1,\ 0.1,$ and 0.01 using the formula

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h}$$

Find the order of approximation.

Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial. Hence, estimate the value of y when x = 2

x	–1	0	1	3
y	–12	–3	4	36

6. Approximate the second order derivative of $f(x) = e^{-x}$ at $x_0 = 0$, taking $h = 1, 0.1,$ and 0.01 by using the formula

$$f''(x_0) \approx \frac{f(x_0 - h) - 2f(x_0) + f(x_0 + h)}{h^2}$$

Find the order of approximation.

Approximate the solution of the initial value problem in 5 steps using Euler's method.

$$\frac{dy}{dx} = \frac{e^x}{y}, y(0) = 1, 0 \le x \le 2.$$

Also find the absolute error at each step given that the exact solution is $y(x) = \sqrt{2e^x - 1}$.