Unique Paper Code : **32351102**

Name of Paper : **BMATH102- Algebra**

Name of Course : **CBCS B.Sc.(H) Mathematics**

Semester : **I**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Solve the following equations

(i) $x^4 - 6\,x^3 + 26\,x^2 - 96\,x + 160 = 0$, given that sum of two of the roots is 6.

(ii) $x^4 - 8\,x^3 + 31\,x^2 - 106\,x + 34 = 0$, given that $3 - \sqrt{7}$ is a root of the equation.

(iii) $x^3 - 24\,x^2 - 128\,x + 2048 = 0$, given that it has an integral root.

2. Find the polar representation of the complex number, $z = \frac{(1+i)^8\ (\sqrt{3}+i)^5}{(-1-i\sqrt{3})^8}$ and determine $|z|$, arg z, arg $\bar{z}$ and arg$(-z)$.

Solve the equation: $z^5 = 8\left(i\sqrt{3} - 1\right)$.

3. Define a relation $\sim$ on the set of integers $\mathbb{Z}$ by $u \sim v$ if and only if $5u + 3v$ is a multiple of 8. Prove that $\sim$ is an equivalence relation on $\mathbb{Z}$. Find the equivalence class of integers 0, 2 and 3. Determine the quotient set for this equivalence relation.

Consider a real function $f : \{u \in \mathbb{R}: u \neq -3/5\} \to \mathbb{R}$ defined by $f(u) = \frac{1}{5u+3}$. Show that f is one-one. Find the range of the function f and determine f^{-1}, if it exists.

4. Let S be an infinite set and let x, y be elements not in S. Prove that S and $S \cup \{x, y\}$ are the sets of same cardinality. Deduce that the intervals (2,3) and [2,3] have the same cardinality.

Evaluate 54^{25} (mod 11).

5. Consider the following system of linear equations:

$$x + 9y - z = 27$$
$$x - 8y + 16z = 10$$
$$2x + y + 15z = 37.$$

Write the matrix equation $Ax = b$ and the corresponding vector equation for the above system of linear equations. Find the general solution of the above system of equations by reducing the augmented matrix to row reduced echelon form. Deduce the general solution in parametric vector form for the homogeneous system $Ax = 0$. Find a basis and the dimension for the null space of A.

6. Find the characteristic polynomial, the eigen values and the corresponding eigen vectors and eigen spaces of the following matrix:

$$A = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ -3 & 4 & 6 \end{bmatrix}.$$