Q1. (a) Assume that a Sun like star splits into 10 smaller identical stars. Assume that the total mass is conserved and all the smaller stars have the same surface temperature and densities as the Sun. Compute the combined absolute magnitude of 10 stars if none of the stars obstruct each other light. The absolute magnitude of Sun like star is, $M = +4.7$.  (b) Show that, for two stars (A and B) orbiting in circular orbits around a common centre of mass, the period ($P$) is related to the masses of the two stars ($m_A$ and $m_B$) and the distance between the centres of the two stars ($r$) by  
\[ G \frac{P^2}{2} (m_A + m_B) = 4\pi^2 \frac{r^3}{2} \]

Q2. Define limiting magnitude of an optical instrument. Prove that for a telescope of aperture D (in mm), the magnitude limit in the visible range is given by:

\[ m_v \sim 2 + 5 \log_{10} (D) \]

Assume that the human eye has a limiting magnitude of +6 and the diameter of the pupil of human eye is 7 mm. Why have CCDs replaced photographic films for recording astronomical images?
Q3. Assume that a galactic rotation curve of a galaxy is described by the following function $v(r) = a r^\beta$, where $a$ and $\beta$ are constants, and $r$ is measured radially from the centre of the galaxy. How much mass $M(r)$ is enclosed within the volume of radius $r$? Find the density distribution $\rho(r)$ followed by such a galaxy. Plot the velocity curve $v(r)$ as a function of $r$ for $\beta = -1/2, 0, 1$.

Q 4. The general intensity profile of a galaxy is given as:

$I(r) = I(0) \exp(-r/b)^{1/n}$, where “$b$” is the scale length and $I(0)$ is the central intensity and $n$ is the shape parameter. Find the total light emitted by the galaxy in terms of $I(0)$, $n$ and $b$.

Q 5. Explain how the size and distance of a star which belongs to the main sequence can be estimated using Hertzsprung Russell (HR) diagram. If a star has a parallax of 0.05” and apparent bolometric magnitude is $m_{bol} = 4$, find the luminosity of the star in terms of solar luminosity.

Q6. Discuss the coordinate system in which coordinates of the celestial body do not change with time and position of the observer on the Earth. Suppose an astronomer wants to point a telescope to a celestial body whose azimuth is 37° and altitude is 58°. Describe the procedure which should be adopted by an astronomer to view this celestial object.

Useful constants:

$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$

$k_B = 1.38 \times 10^{-23} \text{ J/K}$

$\sigma = 5.67 \times 10^{-8} \text{ W m}^2 \text{ K}^{-4}$

$M_{\text{sun}} = 1.99 \times 10^{30} \text{ kg}$

$R_{\text{sun}} = 6.96 \times 10^8 \text{ m}$

$L_{\text{sun}} = 3.86 \times 10^{26} \text{ W}$

1 A. U. = 1.5 x 10^{11} m