Sr. No. of Question Paper	:	2
Unique Paper Code	:	32227502
Name of the Paper	:	Advanced Mathematical Physics (DSE Paper)
Name of the Course	:	B. Sc. (Hons) Physics (CBCS)
Semester	:	V

Duration : 3 Hours Maximum Marks: 75

Attempt any ***four*** questions. All questions carry equal marks.

1. (a) Consider a set of 2 × 2 matrices, S = $\{I, A, B, C\}$,

where, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$;

$C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $i = \sqrt{-1}$.

Determine whether S forms a group w. r. t. matrix multiplication. (2.75)

(b) Let F be the transformation defined by F(x, y) = (3x + 2y, 5y – 1).

Determine whether F is linear and singular. (2, 2)

(c) Consider **T: R**^{2} → **R**^{2}, defined by T(1, 1) = (3, 2) and T(0, 1) = (–2, 1).

Find T(a, b) and $T^{-1}(14, 7)$. (8, 4)

2. (a) If α, β, γ are linearly independent vectors. Determine whether or not the vectors $\alpha, \alpha + \beta, \alpha + \beta + \gamma$ are linearly independent. (5)

(b) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

Can A be diagonalized? If yes, find a diagonalizing matrix P and verify that P diagonalizes A. (13.75)

3. (a) Solve the coupled differential equations

$$\dot{y} = y + z$$

$$\dot{z} = 4y + z \text{ ; where, } y(0) = z(0) = 1.$$ (13.75)

(b) If H is a square matrix, prove that $\det\left(e^{H}\right) = e^{Tr(H)}$. (5)

4. (a) If $\Delta(a) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, prove that

$$\Delta(a) = a_{1h}\, a_{2k}\, a_{3u} \in_{hku}$$

$$\in_{pcm}\ \Delta(a) = a_{ph}\, a_{ck}\, a_{mu} \in_{hku}$$

and $6\,\Delta(a) = a_{ph}\, a_{ck}\, a_{mu} \in_{pcm} \in_{hku}$ (5, 5, 3)

(b) Show that

$\in'_{hsu} = \in_{hsu}$, i. e., $\in_{hsu}$ is an isotropic tensor (3)

and $\in_{hku} \in_{pcm} \delta_{kc}\, \delta_{um} = 2\,\delta_{hp}$ (2.75)

5. (a) Using tensors, show that dot product of two vectors is invariant. (3.75)

(b) Using tensors, prove that

(i) $\left(A \times B\right) \times C = B\left(A \bullet C\right) - A\left(B \bullet C\right)$ (7)

(ii) $\nabla \bullet \left(A \times B\right) = \left(\nabla \times A\right) \bullet B - \left(\nabla \times B\right) \bullet A$ (8)

6. (a) If $R_{pk} = \begin{bmatrix} x_2^{\,2} + x_3^{\,2} & -x_1\, x_2 & -x_1\, x_3 \\ -x_2\, x_1 & x_1^{\,2} + x_3^{\,2} & -x_2\, x_3 \\ -x_3\, x_1 & -x_3\, x_2 & x_1^{\,2} + x_2^{\,2} \end{bmatrix}$

then using quotient law **or** otherwise, show that R_{pk} behaves like a tensor of order two if the coordinate system is rotated about some axis by angle θ. (5)

(b) If $d\,s^2 = 7\left(dx^1\right)^2 + 5\left(dx^2\right)^2 + 3\left(dx^3\right)^2 - 4\,dx^1\,dx^3 + 2\,dx^2\,dx^3$,

then find the matrices of g_{ps} and g^{ps}. (3, 4)

(c) Show that $A^{\mu}\, B_{\mu}$ is invariant and $A^{\mu}\, B_{\mu} = A_{\mu}\, B^{\mu}$. (3, 3.75)