1. Solve the following problems as indicated:
   i. Find the orthogonal trajectories of the family of curves: $x^2 - y^2 + 2\rho xy = 1$, where $\rho$ is a parameter.
   ii. Find an integrating factor and solve: $(1 - x^2)y\,dy + 2(y^2 + 4)\,dx = 0$, $y(3) = 0$.

2. Solve the following problems as specified:
   i. Reduce the equation to homogeneous form using the substitution $y = z^2$ and hence solve it:
      $$2x^2y\frac{d^2y}{dx^2} + 4y^2 = x^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}.$$
   ii. Find the complimentary functions for the differential equations:
      $$\frac{d^2y}{dx^2} + y = x^2,\quad 2\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 12y = e^x,\quad 16\frac{d^2y}{dx^2} - 24\frac{dy}{dx} + 9y = \sin x.$$
   iii. Find a second order homogeneous linear ordinary differential equation having $x^{-3}$ and $x^{-5}$ as its solutions. Also use Wronskian to show linear independence or dependence of these solutions.

3. Using method of undetermined coefficients, solve the differential equations:
   i. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \cos x$.
   ii. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = x^2$.

4. Find the series solution of the differential equations:
   i. $\frac{d^2y}{dx^2} + 2xy = 0$.
   ii. $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$.

5. Form the partial differential equations by eliminating the arbitrary constants or arbitrary functions from the following surfaces:
   i. $2z = mx^2 + ny^2 + mn$, $m$ and $n$ are arbitrary constants.
   ii. $2z = a + (x + by)^2$, $a$ and $b$ are arbitrary constants.
   iii. $z = x + y + f_1(cx + y) + f_2(cx - y), c(\neq 0)$ is a fixed constant, $f_1$ and $f_2$ are arbitrary functions.

6. Identify the equation which is parabolic by nature. Reduce that equation to canonical form and hence solve that equation.
   i. $x^2u_{xx} - y^2u_{yy} - 2yu_y + \sin x\,u_x = 0, x \neq 0, y \neq 0$. 

Attempt any four questions. All questions carry equal marks.
ii. \[ 4y^2u_{xx} - 3xyu_{xy} + x^2u_{yy} + xu_x + yu_y = 0, x \neq 0, y \neq 0. \]

iii. \[ y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} - \frac{y^2}{x}u_x - \frac{x^2}{y}u_y = 0, x \neq 0, y \neq 0. \]