Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32355301_OC**
Name of Paper	: **GE-3 Differential Equations**
Semester	: **III**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Solve the following problems as indicated:

i. Find the orthogonal trajectories of the family of curves: $x^2 - y^2 + 2\rho xy = 1$, where ρ is a parameter.

ii. Find an integrating factor and solve: $(1-x^2)ydy + 2(y^2+4)dx = 0,\ y(3) = 0$.

2. Solve the following problems as specified:

i. Reduce the equation to homogeneous form using the substitution $y = z^2$ and hence solve it:

$2x^2y\frac{d^2y}{dx^2} + 4y^2 = x^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$.

ii. Find the complimentary functions for the differential equations:

$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x^2$, $2\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 12y = e^x$, $16\frac{d^2y}{dx^2} - 24\frac{dy}{dx} + 9y = \sin x$.

iii. Find a second order homogeneous linear ordinary differential equation having x^{-3} and x^{-5} as its solutions. Also use Wronskian to show linear independence or dependence of these solutions.

3. Using method of undetermined coefficients, solve the differential equations:

i. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \cos x$.

ii. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = x^2$.

4. Find the series solution of the differential equations:

i. $\frac{d^2y}{dx^2} + 2xy = 0$.

ii. $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$.

5. Form the partial differential equations by eliminating the arbitrary constants or arbitrary functions from the following surfaces:

i. $2z = mx^2 + ny^2 + mn$, *m* and *n* are arbitrary constants.

ii. $2z = a + (x+by)^2$, *a* and *b* are arbitrary constants.

iii. $z = x + y + f_1(cx+y) + f_2(cx-y), c(\neq 0)$ is a fixed constant, f_1 and f_2 are arbitrary functions.

6. Identify the equation which is parabolic by nature. Reduce that equation to canonical form and hence solve that equation.

i. $x^2u_{xx} - y^2u_{yy} - 2yu_y + \sin x\, u_x = 0, x \neq 0, y \neq 0$.

ii. $4y^2u_{xx} - 3xyu_{xy} + x^2u_{yy} + xu_x + yu_y = 0, x \neq 0, y \neq 0$.

iii. $y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} - \frac{y^2}{x}u_x - \frac{x^2}{y}u_y = 0, x \neq 0, y \neq 0$.