Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32357501**
Name of Paper	: **DSE-I NUMERICAL METHODS**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Consider the equation $2x - \log_{10} x - 7 = 0$ on]3.78,3.79[. Apply bisection method to find an approximate root of the equation. Do two iterations.

Find an approximate root of the equation $f(x) = x \log_{10} x - 1.2 = 0$ using Regula-Falsi method. Do two iterations.

Approximate the second order derivative of $f(x) = e^x$ at $x_0 = 0$, taking $h = 1, 0.1$ and 0.01 by using the formula

$$f''(x_0) \approx \frac{f(x_0-h)-2f(x_0)+f(x_0+h)}{h^2}.$$

2. Find the maximum value of the step size h that can be used in the tabulation of $f(x) = \sin 5x$ on $[1,2]$ so that the error in the linear interpolation of $f(x)$ is less than 5×10^{-4}.

The equation $f(x) = x^4 - 4x^2 + 4 = 0$ has a double root at $x = \sqrt{2}$. Starting with $x_0 = 1.5$, compute two successive approximations to the root by Newton-Raphson method.

Show that the equation $log_e x = x^2 - 1$ has exactly two real roots, $\alpha = 0.45$ and $\beta = 1$. Determine for which initial approximation x_0, the iteration $x_{i+1} = \sqrt{1 + \log_e(x_n)}$ converges to α or β.

3. Compute T_{jac} for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 4 & 2 \\ 0 & 1 & 6 \end{bmatrix}$. Also determine the spectral radius of the above iteration matrix.

Solve the following system of equations using SOR iteration method

$$\begin{aligned} 2x_1 - x_2 + x_3 &= 0 \\ -x_1 + 4x_2 + 2x_3 &= 4 \\ x_1 + 2x_2 + 6x_3 &= 5. \end{aligned}$$

Take $w = 0.9$ with $X^{(0)} = [0, 0, 0]^T$ and iterate three times.

4. Find an LU decomposition of the matrix $A = \begin{bmatrix} 6 & -2 & 3 \\ 3 & -1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ and use it to solve the system $AX = [5, 6, 5]^T$.

Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial. Hence, estimate the value of y when $x = -3$.

x	-1	0	1	3
y	-121	-60	-29	-33

Hence, estimate the value of y when $x = 2$.

5. Approximate the derivative of $f(x) = \sin 2x$ at $x_0 = \pi$, taking $h = 1, 0.1 \; and \; 0.01$. Using the formula

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h}$$

Find the order of approximation.

Solve the following system of equations using Gauss Seidel iteration method

$$\begin{aligned} 2x_1 - x_2 + x_3 &= -1 \\ -x_1 + 4x_2 + 2x_3 &= 3 \\ x_1 + 2x_2 + 6x_3 &= 5. \end{aligned}$$

Take $X^{(0)} = [0, 0, 0]^T$ and iterate three times.

6. Find the order of approximation. The following are the five successive iterations obtained by secant method to find the root

$\alpha = -2$ of the equation $x^3 - 3x + 2 = 0$,

$x_1 = -2.6, x_2 = -2.4, x_3 = -2.106598985, x_4 = -2.022641412, x_5 = -2.000022537.$

Compute the asymptotic error constant and show that $\varepsilon_5 \approx \frac{2}{3}\varepsilon_4$.

Approximate the solution of the initial value problem in 5 steps using Euler's method

$$\frac{dy}{dx} = \frac{x}{y}, y(0) = 1, 0 \leq x \leq 5.$$

Also find the absolute error at each step given that the exact solution is $y(x) = \sqrt{x^2 + 1}$.