

Sr. No. of Question Paper : 2  
 Unique Paper Code : 32221301-OC  
 Name of the Paper : Mathematical Physics II  
 Name of the Course : B. Sc. (Hons) Physics (CBCS)  
 Semester : III

**Duration : 3 Hours**

**Maximum Marks: 75**

Attempt any **four** questions. All questions carry equal marks.

1. Given,  $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \frac{\pi x}{4}, & 0 < x < \pi \end{cases}$  with  $f(x+2\pi) = f(x)$

(a) Find Fourier Series and hence prove that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$$

(10.75, 3)

(b) Using Parseval's Identity, prove that:

$$1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \dots = \frac{\pi^4}{96} \quad \text{Given} \quad \sum_{n=0}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

(5)

2. (a) Find Fourier Cosine Series for the function

$$f(x) = x(\pi - x), \quad 0 < x < \pi$$

Hence, prove that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$$

(10.75)

(b) Using Beta and Gamma functions, evaluate:

$$\int_0^{\pi/2} \sqrt{\tan \theta} \, d\theta \quad \text{and} \quad \int_0^{\infty} \frac{x \, dx}{x^6 + 1}$$

(4, 4)

3. (a) Identify and name the singularities of the following differential equation:

$$x^2(1-x^2) \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} + 4y = 0$$

(6)

(b) Solve the following differential equation using Frobenius method:

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} - xy = 0$$

(12.75)

4. (a) Prove the Recurrence Relation for Bessel Polynomial:

$$n J_n(x) + x J_n'(x) = x J_{n-1}(x)$$

(6.75)

(b) Show that:  $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left( \frac{\sin x - x \cos x}{x} \right)$  (6)

(c) Using Generating Function, show that:

$$J_n(u+v) = \sum_{s=-\infty}^{\infty} J_s(u) J_{n-s}(v) \quad (6)$$

5. (a) Expand  $f(x) = x^3$  in a series of Legendre Polynomial,

and hence evaluate  $\int_{-1}^1 x^3 P_3(x) dx$  (5, 6)

(b) Write the Rodrigue's formula for Legendre polynomials and hence find

$$P_1(x), P_2(x) \text{ and } P_3(x) \quad (7.75)$$

6. (a) Using the method of separation of variables, solve the following differential equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (9)$$

subject to the conditions  $u(0, y) = u(1, y) = u(x, 0) = 0$  and  $u(x, a) = F(x)$ .

(b) Solve the one dimensional wave equation under the following conditions:

$$u(0, t) = u(1, t) = 0$$

$$u(x, 0) = \begin{cases} x & \text{for } 0 \leq x < 1/2 \\ 1-x & \text{for } 1/2 < x \leq 1 \end{cases}, \quad (9.75)$$

and  $\frac{\partial u}{\partial t} \Big|_{t=0} = 0$ , where  $u(x, t)$  is the displacement of the vibrating string.