**(a) Name of the Department:** PHYSICS DEPARTMENT

**(b) Name of the Course:** B.Sc. Hons.–CBCS_Core

**(c) Name of the Paper:** Quantum Mechanics and Applications

**(d) Semester:** V- Semester

**(e) Unique Paper Code:** 32221501

**(f) Question paper Set number:** SET 1

**Time Duration: 3 hours** **Total Marks: 75**

***Attempt four questions out of six. Each question carries equal marks.***

1. (a) A particle of mass 'm' is in the state

$$\Psi(x,t) = A e^{-a[(mx^2/\hbar)+it]}$$

where A and a are positive real constants.

(i) Find A. (Use: $\int_0^\infty e^{-ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{a}}$)

(ii) For what potential energy function V(x) does $\psi$ satisfy the Schrodinger equation.

(iii) Calculate the expectation values of x, $x^2$, p and $p^2$. Find the uncertainties in position and momentum, $\sigma_x$ and $\sigma_p$ respectively. Is their product consistent with uncertainty principle?

(Use: $\int_0^\infty x^2 e^{-ax^2} dx = \frac{\sqrt{\pi}}{4\sqrt{a^3}}$)

(b) State with reason which of the following wave functions are physically acceptable solutions of the Schrodinger wave equation:

(i) tan (x), (ii) exp($-x^2/2$)

2+2+12.75+2

2. (a) Show that for a time independent potential V(x), the Schrodinger equation may be solved for the time dependence by separation of variables and reduced to the time independent Schrodinger equation. If $\psi_n(x)$ are the bound state wave functions corresponding to the energy eigen values $E_n$ , write down the solution of the time dependent Schrodinger equation as a linear combination of these wave functions.

(b) Let $E_n$ denote the bound-state energy eigenvalues of a 1-D system and $\psi_n(x)$ denote the corresponding energy eigenfunctions. Let $\Psi(x,t)$ be the wavefunction of the system normalized to unity, and suppose that at t = 0, it is given by

$$\Psi(x,t=0)=\frac{1}{\sqrt{2}}e^{i\alpha_1}\psi_1(x)+\frac{1}{\sqrt{3}}e^{i\alpha_2}\psi_2(x)+\frac{1}{\sqrt{6}}e^{i\alpha_3}\psi_3(x)$$

where the $\alpha_i$ are real constants.

(i) Write down the wave function $\Psi(x,t)$ at time t.
(ii) Find the probability that at time 't', a measurement of energy of the system gives the value $E_2$.
(iii) Do $\langle x \rangle$ and $\langle p_x \rangle$ vary with time? Explain.

(c) Prove that the following operators are Hermitian.
(i) $\hat{x}$ (ii) $\hat{p}$

7+2+1.75+4+4

3. (a) Solve the Schrodinger equation for a particle having energy $E<V_0$ for a square well potential of finite depth Vo. Show graphically that that there exists at last one bound state.

(b) Consider the classical Hamiltonian for 1-D system of two particles of masses $m_1$ and $m_2$ subjected to a potential that depends only on distance between them $(x_1 - x_2)$,

$$H=\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}+V(x_1-x_2)$$

Write the Schrodinger equation using the new variables x and X, where

$$x=x_1-x_2 \text{ (relative distance)}$$

$$X=\frac{m_1x_1+m_2x_2}{m_1+m_2} \text{ (center of mass)}$$

(c) If $\Psi(x,t)$ is a solution of the Schrodinger equation for a free particle of mass 'm' in 1-D, and

$$\psi(x,0)=Ae^{-x^2/a^2}$$

Find the wave function in momentum space and $\Psi(x,t)$.

(Use: $\int_{-\infty}^{\infty} e^{-\alpha^2(x+\beta)^2} dx = \frac{\sqrt{\pi}}{\alpha}$) 10+4.75+4

4. (a) Find the allowed energies of the half-harmonic oscillator,

$$V(x) = \begin{cases} \left(\frac{1}{2}\right) m\omega^2 x^2, for\ x > 0 \\ \infty, for\ x < 0 \end{cases}$$

(b) A particle of mass 'm' is confined to a 1-D line of length L. From argument based on the wave interpretation of matter, show that the energy of the particle is quantized.

(c) Determine the probability of finding a particle of mass 'm' between x = 0 and x = L/10, if the particle is described by the normalized wave function,

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L} \text{ for } 0 \le x \le L$$ 8+6+4.75

5. (a) A radial function in spherical polar coordinates is

$$R_n(r) = C e^{-r/2} U_n(r)$$

where C is a normalization constant. Discuss the physical acceptability of $R_n(r)$ if $U_n(r)$ behaves as, (i) $1/r^2$ for small values of r and as a polynomial in r otherwise, and (ii) polynomial in r of degrees more than 3 for all values of r.

(b) Starting from the Schrodinger equation for hydrogen atom in spherical polar coordinates, split the equation into three parts. Obtain the solution for radial wave equation. 4+14.75

6. (a) Discuss the significance of Stern-Gerlach experiment. How does it lead to the space quantization due to spin?
(b) On the basis of quantum theory, explain the normal Zeeman effect.
(c) The quantum numbers of 2 electrons in a 2 valence electron atom are:
(i) $l_1 = 3, s_1 = 1/2$ (ii) $l_2 = 2, s_2 = 1/2$

Assuming L-S coupling, find the possible values of L and J. 8+6.75+4