Sr. No. of Question Paper	:	1
Unique Paper Code	:	32221301
Name of the Paper	:	Mathematical Physics II
Name of the Course	:	B. Sc. (Hons) Physics (CBCS)
Semester	:	III

Duration : 3 Hours **Maximum Marks: 75**

Attempt any ***four*** questions. All questions carry equal marks.

1. Given, $f(x)=\begin{cases} x+1, -1<x<0 \\ x-1, 0<x<1 \end{cases}$, $f(x+2)=f(x)$

(a) Find Fourier Series representation for $f(x)$ and hence show that

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\ldots=\frac{\pi}{4}$$ (9.75, 3)

(b) Using Parseval's Identity, prove that:

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\ldots=\frac{\pi^2}{6}$$ (6)

2. (a) Given $f(x)=x^2, 0<x<\pi$, find even periodic extension of $f(x)$,

write down its Fourier Cosine Series and hence show that:

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots=\frac{\pi^2}{12}$$ (10.75)

(b) Using Beta and Gamma functions, evaluate:

$$\int_0^{\pi/2} \tan^4\theta\, d\theta \quad \text{and} \quad \int_0^{\infty} \frac{x^2\,dx}{x^4+1}$$ (4, 4)

3. (a) Find the singular points of the following differential equation and classify them as regular or irregular singular points.

$$x^2(x-2)^2\frac{d^2y}{dx^2}+2(x-2)\frac{dy}{dx}+(x+1)y=0$$ (4.75)

(b) Solve the following differential equation using Frobenius method:

$$(x^2+1)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-y=0$$ (14)

4 (a) The general form of Bessel differential equation is given as:

$$x^2\frac{d^2 y}{dx^2} + (1-2a)x\frac{dy}{dx} + \left[a^2 - c^2 p^2 + b^2 c^2 x^{2c}\right]y = 0$$

Reduce this general form to the standard form

$$t^2\frac{d^2 z}{dt^2} + t\frac{dz}{dt} + (t^2 - p^2)z = 0$$

where, $y = x^a z$ and $t = bx^c$. (10)

(b) Prove that: $\left[J_{1/2}(x)\right]^2 + \left[J_{-1/2}(x)\right]^2 = \dfrac{2}{\pi x}$ (8.75)

5 (a) Write the generating function of Legendre Differential equation and hence find

$P_1(x), P_2(x)$ and $P_3(x)$ (7.75)

(b) Expand the function $f(x) = x^4 - 3x^2 + x$ in a series of the form

$\sum_{k=0}^{\infty} A_k P_k(x)$, where A_k are real constants. (11)

6 (a) Using the method of separation of variables, solve the following differential equation:

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u$$

with $u(x,0) = 3e^{-5x} + 2e^{-5x}$. (6.75)

(b) By solving Laplace equation in 2D, determine the steady state temperature of a thin rectangular plate bounded by the lines $x=0$, $x = w$, $y=0$, $y = b$, assuming that the edges $x=0, x=w, y=0$ are maintained at zero temperature and the edge $y=b$ is maintained at steady state temperature $F(x)=x$ (12)