Unique Paper Code   : **42351101**

Name of the Paper   : **Calculus and Matrices**

Name of the Course   : **B.Sc. (Math. Sci.)-I, B.Sc. (Phy. Sci.)-I,**
                              B.Sc. (Life. Sci.)-I

Semester   : **I**

Duration   : **3 Hours**

Maximum Marks   : **75**

Attempt any four questions. All questions carry equal marks.

1. Check whether the set $\{(0,2,0),(3,0,-1),(-1,1,0)\}$ is linear independent or not. Is the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(x,y) = (2x+y,-y)$ linear? Sketch the image of the unit square with vertices $(0,0),(0,1),(1,1),(1,0)$ under the given transformation. Find a matrix representation for T.

2. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation satisfying $T(1,1) = (1,7)$ and $T(1,2) = (2,6)$. Find a matrix representation for T and determine $T(1,3)$.

   Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{bmatrix} 5 & -1 \\ 0 & 2 \end{bmatrix}$.

   An amount of 20 ml of a medicine is injected into a patient's body. Half the amount of the medicine is absorbed by the patient body in 12 hours. How long will it take for the patient to absorb 70% of the medicine?

3. Solve, if consistent, the following system of linear equations using elementary row operations
$$x + y + z = 5$$
$$2x + 3y + 4z = 16$$
$$y - 4z = -12.$$

   Reduce the following matrix A to triangular form using elementary row operations and also determine its rank
$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & -17 \\ 2 & 11 & -8 \end{bmatrix}.$$

   Is $W = \{(2x, y+1, 0) : x, y \in \mathbb{R}\}$ a subspace of $\mathbb{R}^3$? Justify your answer.

4. Discuss the convergence of the sequences $a_n = \sqrt{1 + (-\frac{1}{2})^n}$ and $b_n = 5^{n/(n+1)}$.

   Find the nth derivative of $y = 2e^x \sin x \cos 2x$.

   Also sketch the graph of the functions
$$f(x) = 5 - |x+3| \text{ and } f(x) = 1 + 3e^{-2x}$$
   mentioning the transformations used at each step.

5. Find the Taylor's polynomial of order 4 generated by the function $f(x) = \sin 3x$ at $x = \frac{\pi}{3}$.

If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

Verify that the function $u(x, t) = \sin(x + 3t) + \cos(x + 3t)$ satisfies the Wave Equation.

6. Find the polar representation of the points $z_1 = -1 - i$, $z_2 = -1 + i\sqrt{3}$ and $z_1 z_2$.

Form an equation in lowest degree with real coefficients which has $3 - i$, $1 + 3i$ as two of its roots.

Solve the equation $z^3 - 1 + i = 0$.

Find the equation of the straight line joining the points
$$z_1 = -1 - i \quad \text{and} \quad z_2 = -1 + i\sqrt{3}.$$