Attempt any four questions. All questions carry equal marks.

1. Determine the constant $A$ such that the differential equation

$$(Ax^2 y + 2y^2)dx + (x^3 + 4xy)dy = 0$$

is exact and solve the resulting exact equation.

Solve the following initial value problems:

i) $\frac{dy}{dx} + \frac{4}{x}y = x^3y^2, \quad y(2) = -1, \quad x > 0$.

ii) $(3xy + y^2)dx + (x^2 + xy)dy = 0, \quad y(1) = -1$.

2. Find the orthogonal trajectory of the family $y = c \sin x$ that passes through the point $(2\pi, 2)$. Also find the family of oblique trajectory that intersects the family of circles $x^2 + y^2 = c^2$ at an angle $\frac{\pi}{4}$.

Show that the family of confocal conics $\frac{x^2}{\lambda + a^2} + \frac{y^2}{\lambda + b^2} = 1$, where $a$ and $b$ are fixed constants and $\lambda$ is the parameter, is self orthogonal.

3. Show that the set $\{1, x, x^2\}$ of functions forms a basis for the solution set of a differential equation. Also, find such a differential equation.

Find the general solution of the second order equation $t^2y''' + 2ty'' - 2y = 0$ given that $y_1(t) = t$ is a solution. Also solve the initial value problem

$$y''' + 3y'' - 4y = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = \frac{1}{2}.$$

4. Find the general solution of the following differential equations:

i) $\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 2x^2 + 3e^{2x}$.

ii) $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = e^x \log x$.

iii) $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log x$. 

5. Find the partial differential equation arising from the equation \( ax^2 + by^2 + z^2 = 1 \), where 
\[ z = z(x, y). \]

Find the general solution of the linear partial differential equation
\[ (y + xu)p - (x + yu)q = x^2 - y^2. \]
Using \( v = \ln u \) and \( v = f(x) + g(y) \), solve the equation
\[ y^2 u_x^2 + x^2 u_y^2 = (xyu)^2, \quad u(x, 0) = e^{x^2}. \]

6. Reduce the following equations to canonical form and hence find their solutions
i) \( u_x - yu_y = 1 + u. \)
ii) \( yu_{xx} + (x + y)u_{xy} + xu_{yy} = 0, \quad y \neq x. \)
iii) \( u_{xx} - 4u_{xy} + 4u_{yy} = 0. \)