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| Name of Course | : **Generic Elective** |
| Unique Paper Code | : **32355301** |
| Name of Paper | : **GE-3 Differential Equations** |
| Semester | : **III** |
| Duration | : **3 hours** |
| Maximum Marks | : **75 Marks** |

*Attempt any four questions. All questions carry equal marks.*

**1.** Determine the constant $A$ such that the differential equation

$$(Ax^2y + 2y^2)dx + (x^3 + 4xy)dy = 0$$

is exact and solve the resulting exact equation.

Solve the following initial value problems:

i) $\frac{dy}{dx} + \frac{4}{x}y = x^3y^2, \quad y(2) = -1, \ x > 0.$

ii) $(3xy + y^2)dx + (x^2 + xy)\mathrm{d}y = 0, \ y(1) = -1$ .

**2.** Find the orthogonal trajectory of the family $y = c\sin x$ that passes through the point $(2\pi, 2)$. Also find the family of oblique trajectory that intersects the family of circles $x^2 + y^2 = c^2$ at an angle $\frac{\pi}{4}$ .

Show that the family of confocal conics $\frac{x^2}{\lambda + a^2} + \frac{y^2}{\lambda + b^2} = 1$, where $a$ and $b$ are fixed constants and $\lambda$ is the parameter, is self orthogonal.

**3.** Show that the set $\{1, x, x^2\}$ of functions forms a basis for the solution set of a differential equation. Also, find such a differential equation.

Find the general solution of the second order equation $t^2y'' + 2ty' - 2y = 0$ given that $y_1(t) = t$ is a solution. Also solve the initial value problem

$$y''' + 3y'' - 4y = 0, \ \ y(0) = 1, \ \ y'(0) = 0, \ \ y''(0) = \frac{1}{2}.$$

**4.** Find the general solution of the following differential equations:

i) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + 3e^{2x}.$

ii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x.$

iii) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x.$

**5.** Find the partial differential equation arising from the equation $ax^2 + by^2 + z^2 = 1$, where $z = z(x, y)$.

Find the general solution of the linear partial differential equation

$$(y + xu)p - (x + yu)q = x^2 - y^2.$$

Using $v = \ln u$ and $v = f(x) + g(y)$, solve the equation

$$y^2 u_x^2 + x^2 u_y^2 = (xyu)^2, \; u(x, 0) = e^{x^2}.$$

**6.** Reduce the following equations to canonical form and hence find their solutions

i) $u_x - yu_y = 1 + u.$

ii) $y\, u_{xx} + (x + y)u_{xy} + x\, u_{yy} = 0, \; y \neq x.$

iii) $u_{xx} - 4u_{xy} + 4\, u_{yy} = 0.$