Name of Course	: **B.Sc. (Math. Sci.)-I, B.Sc. (Phy. Sci.)-I, B.Sc. (Life Sci.)-I**
Unique Paper Code	: **42351101**
Name of Paper	: **Calculus and Matrices**
Semester	: **I**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find value of a and b, if possible, that will make the function $f(x)$ continuous everywhere where

$$f(x) = \begin{cases} x^2 + ax - b, & x < 1 \\ 3a - 2b, & 1 \le x \le 2 \\ x^3 - ax^2, & x > 2. \end{cases}$$

Show $\lim_{x\to 2}(x^2 - 3) = 1$ using $\epsilon - \delta$ approach.

Given $g(x) = x^2 + 3x - 1$, find formulas to

(i) Stretch the graph of $g(x)$ vertically by a factor of 3 followed by a shift of 4 units towards left.

(ii) Compress the graph of $g(x)$ horizontally by a factor of 4 followed by a reflection about x–axis.

2. Show that the function $f(x) = x^5 + 2x + 1$ has exactly one zero in $[-1, 1]$.

Find Taylor series for $g(x) = x^3 - 2$ about $x = 1$ assuming the validity of the expansion.

Find the n^{th} derivative of $h(x) = x^2 e^{3x}$.

3. Show that the function $u(x,t) = 2\cos(x - ct)$ is a solution of the wave equation.

Let $f(x,y) = ye^x + x^2 y$. Then

(i) Find the slope of the surface $z = f(x,y)$ in the x–direction at the point $(0, -1)$.

(ii) Find the slope of the surface $z = f(x,y)$ in the y– direction at the point $(0, -1)$.

Check the matrix $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$ for diagonalization by finding its eigenvalues and eigenvectors. If diagonalizable, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

4. Determine the values of k such that the system

$$\begin{aligned} kx + y + z &= 1 \\ x + ky + z &= 1 \\ x + y + kz &= 1 \end{aligned}$$

has (i) no solution, (ii) unique solution, (iii) more than one solution by reducing the augmented matrix to echelon form.

Let T be a linear transformation whose standard matrix is $\begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & -1 & 2 & -1 \\ 1 & -3 & 2 & -2 \end{pmatrix}$.

Does T map $\mathbb{R}^4$ onto $\mathbb{R}^3$? Is T a one-to-one mapping?

Given $\mathbf{A} = \begin{pmatrix} 1 & 2 & 2 \\ 3 & -2 & -1 \\ 2 & -5 & 3 \\ 1 & 4 & 6 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 5 \\ -4 \\ 0 \end{pmatrix}$ let S be defined by $S(x) = Ax$. Find x whose image under S is b. Is x unique?

5. Check whether the set of vectors $\{(1,3,-1,4), (3,8,-5,7), (2,9,4,23)\}$ in $\mathbb{R}^4$ are linearly independent or linearly dependent.

Find the rank of the matrix $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & -3 & -2 & -3 \\ 2 & 3 & -2 & 0 & 4 \\ 3 & 8 & -7 & -2 & -11 \\ 2 & 1 & -9 & -10 & -3 \end{pmatrix}$, using elementary row operations.

Find the polar representation of the following complex numbers $z_1 = 1 + i,\ z_2 = \sqrt{3} - i$. Use it to find $\arg(z_1 z_2)$ and $|z_1 z_2|$.

6. Use De Moivre's theorem to prove that

$$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$$

$$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta.$$

Solve the equation $z^6 + z^3 + 1 = 0$.

Find the equation of the straight line joining the points $z_1 = 3 + 4i,\ z_2 = -2 - i$. Also obtain its complex angular coefficient. Determine if z_1, z_2, z_3 are collinear, where

$$z_1 = 3 + 4i,\ z_2 = -2 - i,\ z_3 = 1 + 2i.$$