SET I

Sr. No. of Question Paper : Your Roll No……

Unique paper Code : 42221101

Name of paper : Mechanics

Name of the Course : B.Sc. (Prog.)_CBCS_New Course

Semester : I

Duration : **3 Hours** Maximum Marks : **75**

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. All questions carry equal marks. Attempt any **four** questions in all.

1. What is the significance of the direction of the gradient of a scalar field? What does the zero value for the divergence of a vector field imply? Give physical examples of gradient, curl and divergence. If $\vec{A} = 4\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$, $\vec{B} = 2\hat{\imath} - 10\hat{\jmath} - 7\hat{k}$ and $\vec{C} = 5\hat{\imath} + 7\hat{\jmath} - 4\hat{k}$, deduce the value of
(i) $(\vec{A} \times \vec{B}).\vec{C}$ (ii) $\vec{A} \times (\vec{B} \times \vec{C})$ (iii) $(\vec{A} \times \vec{B}) \times \vec{C}$
Show that the force $\vec{F} = (2xy + z^2)\hat{\imath} + x^2\hat{\jmath} + 2xz\hat{k}$ is a conservative force.

2. Using the law of conservation of momentum obtain the equation of motion of a rocket in free space. A rocket starts from rest with an initial mass M_o and reaches a final velocity v_f when its mass is M_f, show that $\frac{M_f}{M_o} = e^{-\frac{v_f}{v_{rel}}}$ where v_{rel} is the exhaust velocity. A significant amount of energy in the exhaust gases in any rocket flight is 'lost' to heat and light. Does this have any impact on the final rocket velocity? Give reasons for your answer.

3. State and prove work-energy theorem.
Show that the total energy of a satellite of mass m in a circular orbit of radius R around Earth is $-\frac{1}{2}\frac{GmM_E}{R}$, where M_E is mass of the Earth. Thrusters on a satellite are used to change its orbit from initial radius of 10,000 km to the final radius of 15,000 km. How much work is done by thrusters in changing the orbit? Use $m = 1000\ kg$, $M_E = 6 \times 10^{24}$kg and $G = 6.7 \times 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$.

4. State Newton's law of gravitation. Prove that the angular momentum is conserved under the action of a central force? State Kepler's laws of planetary motion. Orbital period of Saturn is 29 years. How large is its orbit compared to Earth's orbit? Both orbits have negligible eccentricities.

5. What is simple harmonic motion? Derive the equation for simple harmonic motion of mass m attached to a spring of spring constant k and find its solution for arbitrary amplitude and phase. What is the phase difference between displacement and velocity, and between displacement and acceleration? A spring with k = 200 N/m is attached to a mass m = 0.50 kg. The mass is given an initial displacement of 0.015 m and an initial velocity of 0.40 m/s away from the equilibrium position. What is the time period and amplitude of the resultant motion? Write down the expression for the motion of mass, including phase.

6. Derive the formula for the variation of the mass of particle with its speed. Electrons in a particle accelerator move at speed 99 percent of the speed of light. What is the mass of an electron moving with this speed as observed from the accelerator fame? How long would a 1.0 m rod placed in the accelerator in the direction of moving electrons appear in an electron's reference frame?