SET A

Unique Paper Code	: **32221102**	
Name of Paper	: **Mechanics**	
Name of Course	: **B.Sc. Hons. Physics-CBCS_NC_Core**	
Semester	: **I**	
Duration	: **3 Hours**	Maximum Marks: **75**

Answer any **four** of the six questions. Each question carries equal marks.

1. Find the center of mass of a solid cone having radius *R* and height *H*.

   A particle of unit mass moves on the x axis under the influence of a force field having potential $V = 6x(x-2)$.

   i. Determine whether the particle is moving in a conservative force field or not.
   ii. Determine the position(s) of stable equilibrium of the particle.
   iii. Find the frequency of small oscillations of the particle about its equilibrium position.

   A cart sliding with speed v on a frictionless track starts leaking sand at a constant rate. Does the cart's speed increase, decrease or remains the same as it moves along the track? Explain.

2. Two particles of masses $m_1$and m_2 respectively are connected by a rigid massless rod of length a and move freely in a plane. Show that the moment of inertia of the system about an axis perpendicular to the plane and passing through the center of mass is μa^2, where μ is the reduced mass.

   A solid sphere and a hollow sphere of the same radius both start from rest at the top of an inclined plane of angle α and roll without slipping down the incline. Which one gets to the bottom first? Explain. Given the coefficient of friction μ, determine the maximum value that the angle of incline α can take, so that neither of the spheres slip as they roll down.

   A ball of mass $m_1 = 2\ kg$ moving at $10\ m/s$ makes an off-centre collision with a ball of mass $m_2 = 3\ kg$ that is initially at rest. After the collision, m_1 is deflected at an angle of 30^o from its original direction of motion and m_2 is moving at $4\ m/s$.

   i. Find the speed of $m_1$and the direction of m_2 after the collision.
   ii. What is the scattering angle of m_1 after the collision in C-frame?

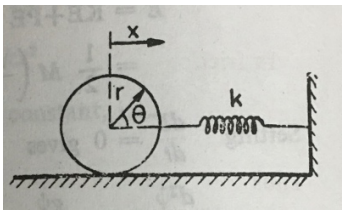
3. Consider a uniform spherical distribution of stars in a galaxy of total mass M and radius R_o. A star of mass M_s at some distance $r < R_o$ from the center will move under the action of a central force whose magnitude depends on the mass included within a sphere of radius r.
   i. What is the force at r ?
   ii. What is the tangential velocity of the star if it moves about the center in a circular orbit?

   A satellite revolves round a planet in an elliptical orbit. Its maximum and minimum distances from the planet are 1.5 x 10^7m and 0.5 x 10^7m respectively. If the speed of the satellite at the farther point is 5 x 10^3 m/s. Calculate the speed at the nearest point.

   Find the condition for which circular orbits are stable if the force function is of the form

   $f(r) = -\frac{k}{r^2} - \frac{\epsilon}{r^4}$.

4. A circular solid cylinder of radius r and mass m is connected to a spring of spring constant k as shown in the figure.



Determine the frequency of horizontal oscillations of the system if the cylinder
   i. Slips on the surface without rolling.
   ii. Rolls on the surface without slipping. (Neglect Friction)

A spring is stretched 5 cm by a force of 50 dynes. A mass of 10g is placed on the lower end of the spring. After equilibrium has been reached, the upper end of the spring is moved up and down so that the external force acting on the mass is given by $F(t) = 20\cos\omega t\,, t \geq 0$.
   i. Find the expression for the position of the mass from its equilibrium position.
   ii. What is the natural frequency of the given spring mass system?
   iii. Explain the role played by $F(t)$ in the motion of mass attached to the spring?
   iv. If $\omega = 1$, then how does the motion of spring-mass get affected? What is this phenomenon called?

5. An ant crawls outward with a constant speed v along the spoke of a wheel that is rotating with constant angular velocity ω about a vertical axis. Explain the various forces experienced by the ant as it crawls along the spoke of the rotating wheel. If the ant starts crawling from the center of the wheel, then how far can it crawl before it starts to slip, given the coefficient of static friction μ_s between the ant and the spoke.

Consider the earth to be sphere of radius R having angular speed ω. Prove that:
   i. Effective value of 'g' at latitude λ is given by $g_{eff} = g_o[1 - (2x - x^2)cos^2\lambda\,]^{1/2}$ where g_o is the true acceleration due to gravity and $x = \omega^2 R/g_o$.
   ii. If $x \ll 1$. then $g_{eff} = g_o - \omega^2 R cos^2\lambda$.

Why is it better in relativity theory to think of spacetime, rather than space and time separately? Give examples to justify your answer. Show that Galilean transformations are not consistent with postulates of the special theory relativity.
For the usual set-up of S' frame and S frame moving along common xx' axes with relative velocity v, the origins coinciding at $t = t' = 0$, if a rod of proper length L oriented parallel to xx' axes move with velocity u along the x axis in S frame, what is the length measured by an observer in S' frame?

6. Derive the relation: $K = (m - m_o)c^2$,where the symbols have their usual meanings. Show that, it reduces to the classical expression of kinetic energy, $K = \frac{1}{2}mv^2$ in the limit $v << c$.

Calculate the speed of an electron having a total relativistic energy of 5 MeV. (Take rest mass of electron as 9.1 x 10^{-31} kg).

A particle having rest mass m_0 collides with an identical particle at rest. The two stick together after the collision. If the particle had speed 0.8c before collision, what is the speed and rest mass of the composite particle?