Name of Course	: **CBCS B.Sc. Mathematical Sciences**
Unique Paper Code	: **42354401_OC**
Name of Paper	: **Real Analysis**
Semester	: **IV**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. State and prove the condition for a sequence of real numbers to have a convergent subsequence. Is the converse true? Give example. Is every Cauchy sequence bounded? If yes, prove it.

2. Using the definition prove the convergence of the following sequences:

(i) $\lim \frac{\text{Cos } n\alpha}{n+1} = 0$ (ii) $\lim \frac{n^3-1}{3+2n^3} = \frac{1}{2}$ (iii) $\lim \frac{1}{(n+1)^2+1} = 0$

Further, find all $x \in \mathbb{R}$ that satisfy the inequality: $|x| + |x+1| < 2$

3. Check the convergence or divergence of the following series

(i) $\sum_{n=1}^{\infty} \sqrt{n^3+1} - \sqrt{n^3-1}$ (ii) $\sum_{n=1}^{\infty} \frac{7^{n+1}}{9^n}$

(iii) $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ (iv) $\frac{1}{e} + \frac{4}{e^2} + \frac{27}{e^3} \ldots$

4. For each of the following, determine whether the series converges absolutely, converges conditionally, or diverges.

(i) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{4n+5}$ (ii) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{n+5}$

(iii) $\sum_{n=1}^{\infty} \frac{\sin n\pi/2}{n^{5/2}}$ (iv) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \log n}$.

5. Check pointwise and uniform convergence of the sequence $< f_n >$ defined as $f_n = nxe^{-nx^2}$ on [0,1] . Show that the series $\sum_{n=1}^{\infty} \frac{\sin(x^2+nx^2)}{n(n+1)}$ is uniformly convergent for all real values of x. Further find the radius of convergence and interval of convergence for the power series

(i) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)}$ (ii) $\sum_{n=0}^{\infty} \frac{n!\,(x+2)^n}{n^n}$.

6. Let f be a function on $[a, b]$ and P be a partition of $[a, b]$ and Q be its refinement, then show that

$$U[Q, f] - L[Q, f] \leq U[P, f] - L[P, f]$$

If f is defined on $[0,1]$ by $f(x) = 2x^2$ $\forall\, x \in [0,1]$, then prove that f is Reimann integrable. Also find the value of integral.

Further show that $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots., -1 \leq x \leq 1$ and hence deduce that $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \ldots.$