

Name of Course	: CBCS B.Sc. Mathematical Sciences
Unique Paper Code	: 42351201_OC
Name of Paper	: C 3-Calculus & Geometry
Semester	: II
Duration	: 3 hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

1. Use (ϵ, δ) definition to find δ such that $\lim_{x \rightarrow 3} (5x - 2) = 13$; $\epsilon = 0.01$.

Examine the continuity of the function $f(x) = \begin{cases} \cos x, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$

at $x = \frac{\pi}{2}$.

Show that the function

$g(x) = \begin{cases} x \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ is continuous everywhere.

2. Discuss the continuity and differentiability of the function $f(x) = (2x - 3)^{\frac{5}{2}}$ at $x = \frac{3}{2}$.

Show that the function $f(x) = e^{-|x|}$ and $g(x) = x + |x|$ are not differentiable at $x = 0$.

Verify the Lagrange's mean value theorem for the function $f(x) = \sqrt{25 - x^2}$ in the interval $[1, 5]$.

3. Find the asymptotes of the curve $xy^2 - x^2y - 3x^2 - 2xy + y^2 + x - 2y + 1 = 0$.

Find the integration of the function $f(x) = \sqrt{x^2 + 4x - 5}$.

Derive the formula for the volume of a sphere of radius r .

4. Describe the graph of the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$.

Find the equation for the ellipse with foci $(0, \pm 6)$ and length of minor axis

16.

5. Trace the conic $16x^2 - 24xy + 9y^2 + 110x - 20y + 100 = 0$ by rotating the coordinate axes to remove the xy -term.

Let $\mathbf{r}(t) = 2t \mathbf{i} + 3t^2 \mathbf{j} + t^3 \mathbf{k}$. Find $\lim_{t \rightarrow 2} \mathbf{r}(t) \cdot (\mathbf{r}'(t) \times \mathbf{r}''(t))$.

Find a vector of length $\sqrt{17}$ that makes an angle of $\pi/6$ with the positive x -axis.

6. Sketch the ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$.

For $\mathbf{A} = 2x^2 \mathbf{i} - 3yz \mathbf{j} + xz^2 \mathbf{k}$ and $\varphi = 2z - x^3y$, find $\mathbf{A} \cdot \nabla \varphi$ and $\mathbf{A} \times \nabla \varphi$ at the point $(2, -1, 1)$.

If $\mathbf{A} = 2yz \mathbf{i} - x^2y \mathbf{j} + xz^2 \mathbf{k}$ and $\varphi = 2x^2yz^3$, find $(\mathbf{A} \cdot \nabla) \varphi$ and $\mathbf{A} \cdot (\nabla \varphi)$.