Name of Course	: **Generic Elective**
Unique Paper Code	: **32355202**
Name of Paper	: **GE-2 Linear Algebra**
Semester	: **II**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. If x and y are vectors in $\mathbb{R}^n$ then prove that $\|2x + 7y\| \le 7\ (\|x\| + \|y\|)$.

Calculate the work done when a force of 95N is exerted on an object in the direction [2, 5, -1] and object travels 50m in the direction of [2, 3, 2].

Let x= [1, 3, 4] and y = [5, -1, 6] be two vectors in $\mathbb{R}^3$. Decompose the vector y into two component vectors in the directions parallel and orthogonal to vector x.

Solve the following system of linear equations using Gauss- Jordan method.

$$x_1 + 2x_2 + 4x_3 = 5$$

$$4x_1 + x_2 + 5x_3 = -3$$

$$6x_1 - x_2 + 15x_3 = 5$$

Also spot the pivot positions.

2. Determine whether the vector V = [1, -1, 2] is in row space of the matrix

$$A = \begin{bmatrix} 4 & -1 & 2 \\ 2 & 3 & 5 \\ 6 & 0 & 9 \end{bmatrix}.$$

If so, express vector V as a linear combination of the rows of matrix A.

Is the matrix $A = \begin{bmatrix} 4 & 0 & -2 \\ 3 & 1 & -3 \\ 2 & 0 & -1 \end{bmatrix}$ diagonalizable ? if yes, find A^5 using diagonalization.

Consider the subset $S = \{x^3 - 1, x^2 - x, x - 1\}$ of $\mathbb{R}^3$. Examine whether S forms a basis for $\mathbb{R}^3$.

3. Let S = $\left\{ \begin{bmatrix} 1 & 6 \\ -2 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 8 \\ 3 & -4 \end{bmatrix} \right\}$ be a subset of 2×2 real matrices. Use the simplified Span Method to find a simplified form for the vectors in Span(S). Is the set linearly independent? Justify.

Using rank, determine whether the homogeneous system of linear equations

$$x + 2y + 3z = 0$$

$$4x + 2y + 6z = 0$$

$$3x + 2y + z = 0$$

has a nontrivial solution or not.

Find the basis and the dimension for the subspace W = $\{ [x, y, z] \in \mathbb{R}^3 : x + 5y + z = 0 \}$.

4. Suppose T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear operator with T ([1,2]) = [1, 1] and T ([2,0]) = [2, -4]. Give a formula for T([x, y]) for any [x, y] ∈ $\mathbb{R}^2$. Also find a basis for ker(T), a basis for range(T) and verify the dimension theorem.

5. Show that the linear transformation T: $\mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

is one-to-one but not onto.

Also check whether the mapping L: $M_{22} \rightarrow M_{22}$ given by L(A) = A + A^T is an isomorphism, where M_{22} is the vector space of all 2×2 matrices over real numbers and A^T is the transpose of A.

6. For the subspace W = {[x, y, z]∈ $\mathbb{R}^3$: 2x+3y+z=0} of $\mathbb{R}^3$, find a basis for the orthogonal complement $W^{\perp}$ and verify that dim(W) + dim($W^{\perp}$) = dim($\mathbb{R}^3$). Decompose v = [6, 10, 5] into $w_1 + w_2$, where $w_1 \in W$ and $w_2 \in W^{\perp}$.