Name of the Course	**: B.Sc. (Hons.) Mathematics CBCS**
Semester	**: IV**
Unique Paper Code	**: 32351402_OC**
Name of the Paper	**: C9 - Riemann Integration and Series of Functions**

Duration: **3 Hours** Maximum Marks: **75**

Attempt any four questions. All questions carry equal marks. All symbols have usual meaning.

1. Calculate the upper Darboux sum and lower Darboux sum of the function

   $f(x) = \frac{1}{x^2}$ *and* $g(x) = xe^x$ on the interval $[1, 2]$ for a partition

   $P = \{1, 1.25, 1.5, 1.75, 2\}$.

   Calculate upper and lower Darboux integral of $g(x) = x^2 + 5$ on $[2,4]$. Is g integrable ?

   Let $f: [a, b] \to R$ be a bounded function. Suppose that there is a partition P of $[a, b]$ such that

   $$L(f,\ P) = U(f,\ P).$$

   Show that f is a constant function.

2. Define $f(x) = x[x]$ on $[0, 4]$. Show that f is integrable, and evaluate $\int_0^4 f(x)dx$. Give an example where the functions f and g are not Riemann integrable, but $f.g$ is integrable. Let f be a continuous function on R and define

   $$F(x) = \int_0^{\frac{x^4}{4}} f(t)dt \quad for\ x \in R$$

   show that F is differentiable on R and compute F'.

3. Examine the convergence of following improper integrals

   $$\int_0^{\infty} e^{-x}(3x + 2)dx \quad and \quad \int_0^{\infty} \frac{dx}{\sqrt{3x^4 + 5x}}$$

   Using the properties of Gamma integral find the value of

   $$\int_0^{\infty} x^5 e^{-4x^2}\, dx$$

4. Let (f_n) be defined by $f_n(x) = 1 - |1 - x^2|^n\ \forall x \in [-\sqrt{2},\ \sqrt{2}\,]$

   Find the pointwise limit of (f_n) on $[-\sqrt{2},\ \sqrt{2}\,]$.

   Does the sequence converge uniformly on this interval? Justify your answer. Show that the sequence (f_n) where $f_n(x) = nxe^{-nx^2},\ x \geq 0$ is not uniformly convergent on $[0,\ 2]$.

   Show that the sequence $\left\{\frac{\sin(n^2x^2+1)}{n(n+1)}\right\}$ converges uniformly on R.

5. Examine the convergence of the series of functions $\sum f_n$ where $f_n(x) = \frac{1}{1+x^n}$ and show that convergence is non uniform in $(1, \infty)$ and is uniform in $[a,\ \infty[,\ a > 1$

Show that $\sum_{n=1}^{\infty} \frac{\cos nx}{n^3}$ converges uniformly on R to a continuous function.

Evaluate the integral $\int_0^1 \sum_{n=1}^{\infty} \frac{x}{(n+x^2)^2} dx$

6. Find the radius of convergence and exact interval of convergence for the following power series

$$\sum_0^{\infty} \frac{4^n}{n5^{n+2}} (x-2)^n \qquad and \qquad \sum_0^{\infty} \left[\frac{3+5(-1)^n}{7}\right]^n x^n$$

Write the power series expansion for the integral of the following function:

$$f(x) = \frac{x^2}{3-x^3} \quad ,$$

given that $\frac{1}{1-x} = \sum_0^{\infty} x^n \quad , x \in]-1,1[$.