

[This question paper contains 2 printed pages.]

**Sr. No. of Question Paper :**

**Your Roll No.....**

Unique Paper Code :32351202\_OC

Name of the Course : **B.Sc. (Hons.) Maths-I**

Name of the Paper : C4-Differential Equations

Semester : II

Duration: 3 Hours

Maximum Marks: 75

**Instruction for Candidates**

- 1) **All questions carry equal marks.**
- 2) **Attempt any four questions.**

1. (a) Find a general solution of the following differential equations.

$$\frac{dy}{dx} = \sqrt{x + y + 1}$$

(b) The half-life of a radioactive cobalt is 6.82 years. Suppose that a nuclear accident has left the level of cobalt radiation in a certain region at 100 times the level acceptable for human habitation. How long will it be until the region is habitable?

2. (a) Use the method of variation of parameters to find a particular solution of the differential equation  $y'' + 4y = x$ .

(b) Find the general solution of  $(4xy') + y^3e^{-2x} = 4xy$

3. (a) A water tank has the shape obtained by revolving the curve  $y = x^{8/3}$  around y-axis. A plug at the bottom is removed at 1:00 P.M., when the depth of the water in the tank is 18 ft. At 3:00 P.M. the depth of the water is 9ft. When will the tank be empty?

(b) Find the general solution of  $xy'' = y'$ .

4. (a) Let  $R(t)$  denote the number of red army and  $B(t)$  denote the number of blue army. Assuming both the armies use aimed fire, formulate the model (a pair of differential equations) and solve them to find the general solution. Also develop a model (a pair of differential equations) for a battle between two armies where both the groups use aimed fire. Assuming that the red army has significant loss due to disease, where the associated death rate (from disease) is proportional to the number of soldiers in that army.
- (b) Find the solution of  $x^2y'' - 6xy' + 3y = 0, y(2) = 3, y'(2) = 1$ .
5. (a) A mass of  $5 \text{ kg}$  is attached to the end of a spring that is stretched  $25 \text{ cm}$  by a force of  $18 \text{ N}$ . It is set in motion with initial position  $x_0 = 0$  and initial velocity  $v_0 = -15 \text{ m/s}$ . Find the amplitude, period and frequency of the resulting motion.
- (b) Use the method of undetermined coefficients to find the general solution of  $y'' - 4y' + 4y = e^{2x}$
6. (a) In a poultry farm, hens are harvested at a constant rate of  $700$  hens per day. The per-capita birth rate for the hen is  $1.4$  hens per week per hen, and the per-capita death rate is  $4.9$  hen per week per hen. Defining each symbol you introduced, write the word equation to describe the rate of change of hen population. Using the above written word equation, obtain the differential equation describing the rate of change of hen population. If the hen population at a given time is  $280000$ . Estimate the number of hen died in one week. Determine if there are any values for which the hen population is in equilibrium.
- (b) Show that the differential equation  $(4x + 3y^2)dx + 2xy dy = 0$  is not exact. Find an integrating factor & solve.