Serial Number of Question Paper	:	**Set B**
Name of the Paper	:	Advanced Mathematical Physics II (DSE)
Name of the course	:	B. Sc. (Hons.) Physics
Semester	:	VI
Unique Paper Code	:	32227625

Duration: 3 + 1 hours Maximum Marks: 75

Attempt any four questions. Each question carry equal marks.

1. Determine the equation of the curve (joining two given points) which produce the least area when revolved about the x-axis lying in the plane of the curve. 10.75

   Obtain the Lagrangian and hence the equation(s) of motion of a simple pendulum (consisting of mass-less rod of length ℓ and a bob of mass m) whose support is rotating with constant angular velocity ω about an axis passing through the equilibrium position. 8

2. Find the Legendre transform, $G(\mathrm{v}_1, \mathrm{v}_2)$ of the function

   $$F(u_1, u_2) = 2u_1^2 + 3u_1 u_2 + u_2^2$$

   where, $\mathrm{v}_1 = \partial F / \partial u_1$ and $\mathrm{v}_2 = \partial F / \partial u_2$. 10.75

   If we have

   $$\mathrm{X} = 2q_1 + 3q_2 + 4q_3 \quad \text{and} \quad \mathrm{Y} = 7p_1^2 + 5p_2^3 + 3p_3^4,$$

   where, q_1, q_2, q_3 are generalized coordinates and p_1, p_2, p_3 are the corresponding generalized momenta. Find the Poisson Bracket of X and Y. 8

3. Elements of permutation group, S_3 are given as:

   $$\mathrm{E} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \quad \mathrm{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \quad \mathrm{B} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix},$$

   $$\mathrm{C} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \quad \mathrm{D} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \quad \mathrm{F} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

   Form the group table of S_3 and find all its non-trivial subgroups. 10.75

   Also, among these subgroups, identify the normal subgroup(s) and cyclic group(s). 8

4. Form a group-table of a multiplicative group G_1 generated from two square matrices A and B, each of order n, subject to the relations

$$A^2 = B^2 = (AB)^2 = E$$

where, E is the identity matrix of order n. 6

Now, consider a set, $G_2 = \{1, -1, i, -i\}$, prove that G_2 forms a group w. r. t. multiplication and check if G_1 and G_2 can be isomorphic to each other or not. 6

(here, $i = \sqrt{-1}$)

Finally, consider a cyclic group of order 4, $G_3 = \{e, a, a^2, a^3\}$. If G_3 has 4 classes, find its irreducible representations. 6.75

5. A card is drawn and replaced in an ordinary 52-card deck. Using Binomial distribution, find the number of times a card must be drawn so that the probability of drawing heart exceeds 0.90. 6

If the probability of success is 0.02, how many trials are necessary for probability of at least one success greater than 0.50. [Use Poisson distribution] 6

Let us define $Z = \dfrac{X - \mu}{\sigma}$

and $P(z_1 \le Z \le z_2) = \displaystyle\int_{z_1}^{z_2} \phi(u)\, du$, where, $\phi(z) = \dfrac{1}{\sqrt{2\pi}} e^{-z^2/2}$

Prove that

$$P(-\infty \le Z \le \infty) = 1$$

$$P(-z_1 \le Z \le 0) = P(0 \le Z \le z_1)$$

$$P(Z \le -z_1) = P(Z \ge z_1)$$ 6.75

6. Counters marked 1, 2, 3 are placed in a bag, one is withdrawn and replaced. The operation being repeated three times, what is the probability of obtaining a total of 7?

8

Find the best values of γ and C so that $PV^{\gamma} = C$ fits the data given in the table below:

10.75

V	05	10	15	20	25
P	850	300	165	100	75