Unique Paper Code: **12271202**

Name of the Paper: **Mathematical Methods for Economics II**

Name of the Course: **B.A. (Hons.) Economics (CBCS Core)**

Semester: **II**

Duration: **3 Hours**

Maximum Marks: **75**

Instructions for the candidates:

1. Answers may be written either in English or in Hindi; but same medium should be used throughout the paper.
2. **There are six questions in all. Attempt any four.**
3. All parts of a question must be answered together.
4. All questions carry equal (18.75) marks.

1. **(a)** $f(x_1, x_2) = \left(\frac{x_1}{x_2^2}\right)^2$. Find the gradient of f at $P = (1, -1)$. Find the directional derivative of f at P in the direction $(h, k) = (\frac{4}{5}, \frac{3}{5})$. Describe the level curves passing through P.

(b) Find the extremum of the function $f(x, y) = 3x^2 + 2y^2 - 4y$ on the closed region bounded below by the parabola $y = x^2$ and above by the line $y = 4$.

(c) What is the general form of the demand equation which has aconstant elasticity of $-n$?

(d) Use the graphical method to solve the problem of maximising: $z = x + y$ subject to the constraints

$$x + 2y \leq 10$$

$$2x + y \leq 16$$

$$-x + y \leq 3$$

$$x, y \geq 0$$

(6, 6.75, 3, 3)

2. **(a)** Consider the production function $Q = F(L, K) = L^\alpha K^\beta$, where $0 < \alpha, \beta < 1$ and $\alpha + \beta < 1$.

(i) Find $\frac{dK}{dL}, \frac{d^2K}{dL^2}$ and comment on their signs.

(ii) Find the elasticity of substitution σ.

(iii) Is the given function Q homogenous? Verify Euler's theorem.

(b) Consider the function $f(x_1, x_2) = x_1^{1/3} x_2^{2/3}$, where$x_1, x_2 > 0$. Is this function concave? Can you also determine if the function is strictly concave or quasi concave? Graph the level curves of the function at height $f(x_1, x_2) = 1$; and $f(x_1, x_2) = 2$?

(c) Find the area of the region bounded by the curve $y = x^2 + x + 1$ and the x-axis between $x = -4$ and $x = 1$.

(d) Write the dual of the following linear programming problem:

$$\text{minimise } z = 4x + 5y \text{ subject to } \begin{cases} 2x + 3y \geq 4 \\ x + 4y \geq 3 \quad x, y \geq 0 \\ 3x + 1 \geq -5 \end{cases}$$

(6, 6.75, 3, 3)

3. **(a)**Suppose the following two equations define y_1 and $y_2$as differentiable functions of x_1, x_2.

$$(y_1 + 2y_2)^5 + x_1 x_2^2 = 2y_1 - x_2 y_2$$

$$(1 + y_1^2)^3 - y_1^2 y_2 = 8x_1 + x_2^5 y_2^2$$

where $y_1 = f(x_1, x_2)$ and $y_2 = g(x_1, x_2)$ in a neighbourhood around $(x_1, x_2, y_1, y_2) = (1,1,1,0)$.

(i) Compute $\frac{\partial y_1}{\partial x_1}, \frac{\partial y_1}{\partial x_2}, \frac{\partial y_2}{\partial x_1}$ and $\frac{\partial y_2}{\partial x_2}$ at $(1,1,1,0)$

(ii) Find an approximate value of $f(1 + 0.1,\ 1 + 0.2)$

(b) Use the method of Langrange multiplier to find the following two -

(i) The minimum value of $x + y$ subject to the constraint $x^{1/2} y^{1/2} = 2$

(ii) The maximum value of $x^{1/2} y^{1/2}$subject to the constraint $x + y = 4$.

Comment on the geometry of each solution.

(d) In measuring the accuracy of forecasts, the negative and positive errors do not necessarily cancel out and it is the absolute error which matters. If forecasts are made for 10 periods into the future and forecast error at time *t* is given by $E(t) = 55 - 16t + t^2$, determine the total absolute error by evaluating the absolute area between $E(t)$ and the *t*-axis from $t = 0$ and $t = 10$.

(6, 7.75, 5)

4. **(a)** **(i)**If $w = f(2x - 3y, 3y - 4z, 4z - 2x)$, what is value of $\left(\frac{1}{2}\frac{\partial w}{\partial x} + \frac{1}{3}\frac{\partial w}{\partial y} + \frac{1}{4}\frac{\partial w}{\partial z}\right)$?

(ii) If $x^y = y^x$, find$\frac{dy}{dx}$ and the elasticity of *y* with respect to *x*.

(b)A consumer's utility function for two goods is $U(x,y) = \frac{1}{\beta}(x^\beta + y^\beta), 0 < \beta < 1$. Find the optimal values of x, y for general values of p_x, p_y and M. Verify the second order conditions. Obtain the associated demand functions of x and y. Check if x and y are substitutes.

(c) Vitamins A, B and C are found in foods F_1 and F_2. One unit of F_1 contains 1 mg. of A, 100 mg. of B and 10 mg. of C. One unit of F_2 contains 1 mg. of A and 10 mg. of B and 100 mg. of C. The minimum daily requirements of A, B and C are 1 mg., 50 mg. and 10 mg., respectively. The cost per unit of F_1 and F_2 are Re. 1 and Rs. 1.50, respectively. Formulate the linear programming problem.

(d) Draw the phase diagram and determine the nature of the possible equilibrium states for the differential equation $\frac{dy}{dt} = y^3 - 2y^2 - 5y + 6$

(6, 6.75, 3, 3)

5. **(a)** A function f of n variables $x = (x_1, x_2, \dots, x_n)$ defined in a cone $K$is homothetic provided that $y \in K, f(x) = f(y), t > 0 \to f(tx) = f(ty)$. Using this definition identify which of the following functions are homothetic:

(i) $f(x_1, x_2) = 5x_1x_2 + 1$

(ii) $g(x_1, x_2) = lnx_1 + lnx_2$

(b) Given $z = x^3y^2 + y^3x$, $x = s^2 + t$, $y = 2s - t$, find $\frac{\partial z}{\partial t}$ when $s = 1$ and $t = 1$.

(c) Examine the concavity/convexity of the following functions:

(i) $f(x,y) = x^\alpha + y^\beta$, where $x, y > 0; 0 < \alpha, \beta < 1$

(ii) $g(x,y) = (x + y)^{1/2}$,where $x, y > 0$

(iii) $z(x,y) = Ax + By + \ln[a^2 - (x^2 + y^2)]$ defined on $S = \{(x,y): x^2 + y^2 < a^2\}$

(c) Obtain the solution of the following macroeconomic model:

$$Y_t = C_t + I_t$$

$$I_t = a(Y_t - Y_{t-1}), a > 0$$

$$C_t = bY_{t-1} + d, b, d > 0$$

Y_t, C_t and I_t are the income, consumption and investment at time t. Find the expression for Y_t in terms of Y_{t-1}in the equilibrium. Solve the corresponding difference equation. What is the condition required for stability?

(6.75, 8, 4)

6. **(a)** The function $C = f(x,t) = te^{-xt}$ gives the concentration of bacteria in the blood (in millions of bacteria/ml) as a function of x,i.e. the dose (in gm) injected of an antibiotic, and the time t (in hours) since the injection.

(i) Find the first order partial derivatives of C w.r.t x and t respectively. Interpret the same and comment on their signs.

(ii) Draw the graphsof the functions $f(x,2)$ and $f(1,t)$ and interpret them.

(b) Derive and classify the extreme points of the following function:

$$f(x_1, x_2) = (x_1^2 - x_2^2)e^{\frac{-(x_1^2+x_2^2)}{2}}.$$

(c) The demand and supply functions in the market for a commodity aregiven by:

$$Q_{dt} = 4 - P_t$$

$$Q_{st} = 1 + 0.5P_{t-1}$$

Q_{dt}andQ_{st} are the quantity demanded and quantity supplied of the commodity at time$t$, and P_t represents the price in time period $t.$ Find the expression for P_t in terms of P_{t-1}inthe equilibrium. Solve the corresponding difference equation. Is the time path of priceoscillatory/non-oscillatory and convergent/divergent?

(6, 7.75, 5)