Set – A

S. No. of Question Paper :

Unique Paper Code : **32227519**

Name of the Paper : **Linear Algebra and Tensor Analysis (DSE)**

Name of the Course : **B.Sc. (Hons) Physics CBCS**

Semester : **V**

Duration: **3 + 1 hours** Maximum marks: **75**

Attempt any four questions. All questions carry equal marks.

1. (a) Assume that $A,\ I-A,\ I-A^{-1}$ are all non-singular matrices, where I is an identity matrix. Show that

$$(I-A)^{-1}+(I-A^{-1})^{-1}=I$$

(b) Determine the values of λ and μ so that the following system has (i) unique solution, (ii) infinitely many solutions and (iii) no solution.

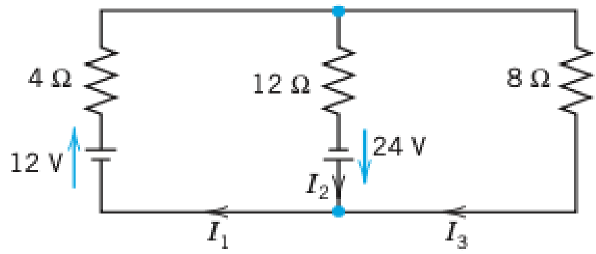
$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

(c) Solve the following system of coupled differential equations using matrix method.

$$y_1'=y_2$$
$$y_2'=y_2$$
$$y_3'=2y_1+y_2-2y_3$$

subject to $y_1(0)=1, y_2(0)=0, y_3(0)=1$ (4.75 + 6 + 8)

2. (a) Consider the circuit given below. Calculate the currents in the circuit using Gauss elimination method.



(b) Given that the three vectors $\alpha_1 = (1,1,0), \alpha_2 = (2,1,0)$ and $\alpha_3 = (1,1,1)$ form a basis for R^3, construct an orthonormal basis for R^3 using Gram Schmidt orthogonalization process.

(c) Calculate the inverse of the following matrix using row reduction method.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

(8.75 + 6 + 4)

3. (a) For what value of k will the vector $(1, -2, k)$ belong to the subspace of $\boldsymbol{R^3}$ spanned by the vectors $(3, 0, 2)$ and $(2, -1, -5)$?

(b) Let V be the vector space of all 2×2 matrices over the real field R. Is the subset W consisting of all matrices A for which $A^2 = A$ a subspace of V?

(c) A linear transformation T on R^3 is defined as:

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x - 3y + 4z \\ 5x - 2y + 2z \\ 4x + 7y \end{bmatrix}$$

Find the matrix representation of T relative to:

i. the standard basis

ii. the basis $\left\{ \alpha_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \alpha_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \alpha_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

iii. matrix of transition from standard basis to α-basis. Also show that the matrices with respect to the two set of bases are related to each other by a similarity transformation.

(4 + 4 + 10.75)

4. (a) Check whether G = $\left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix}; a \text{ is non} - \text{zero real number} \right\}$ is a group with respect to matrix multiplication.

(b) Let V be the space of 2×2 matrices over **R**, and let $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Let T be the linear operator on V defined by $T(A) = MA$. Find the trace of T.

(c) Prove that the components of a strain tensor are given as

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

where u_i is a displacement vector.

A displacement field in a body is given by:

$$u = c(x^2 + 10)$$
$$v = 2cyz$$
$$w = c(-xy + z^2)$$

where $c = 10^{-4}$. Determine the state of strain on an element positioned at $(0, 2, 1)$.

(4 + 6 + 8.75)

5. (a) Show that $L^2 = g_{ij}A^iA^j$ is an invariant. Also show that $L^2 = g^{ij}A_iA_j$.

(b) Prove the following using tensors:

i. $curl(\vec{A} \times \vec{B}) = \vec{A}\, div\, \vec{B} - \vec{B}\, div\, \vec{A} + (\vec{B}.\vec{\nabla})\, \vec{A} - (\vec{A}.\vec{\nabla})\, \vec{B}$

ii. $(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) + (\vec{B} \times \vec{C}) \cdot (\vec{A} \times \vec{D}) + (\vec{C} \times \vec{A}) \cdot (\vec{B} \times \vec{D}) = 0$

(8.75 + 10)

6. (a) Find the condition for two lines to intersect and the equation of the plane in which they lie using tensor notation.

(b) Show that the only isotropic tensor of rank 2 is a scalar multiple of Kronecker delta.

(6.75 + 12)