Name of Course	: **CBCS B.Sc. Phy. Sci.**
Unique Paper Code	: **42354302**
Name of Paper	: **DSC-Algebra**
Semester	: **III**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. (a) Let G = $\{a + b\sqrt{2}\}$ where a and b are rational numbers not both 0. Prove that G is a group under ordinary multiplication. Is this group Abelian? Verify your answer.

(b) Let $A = \begin{bmatrix} 1 & 5 \\ 6 & 3 \end{bmatrix}$. Find A^{-1} in GL(2,$\mathbb{Z}_7$).

2. (a) Prove that if 'a' is any integer relatively prime to n, then $a^{\varphi(n)} = 1$(modulo n) where φ(n) denotes the number of integers (positive) less than n and co-prime to n.

(b) Compute 5^{15}modulo7 and 7^{13}modulo11.

(c) Find all the left cosets of H={1, 11}in U(30).

3. (a) Make a Cayley's Table for the group G of symmetries of a rectangle.

(b) Write all the proper non-trivial subgroups of G.

(c) What is the centre of this group?

4. (a) Show that the set $2\mathbb{Z}_{10}$ ={0, 2, 4, 6, 8}$\oplus_{10}\odot_{10}$ is a ring. By constructing the multiplication table , show that ring has unity.

(b) Show that $\mathbb{Z}_5[i]$ is not an Integral Domain.

(c) Let A and B be ideals of a ring R. If A⋂B={0}, show that ab = 0 when a ∈ A and b ∈ B.

5. (a) Determine whether or not the set{(2, 21, 0), (1, 2, 5), (7, 21, 5)} is linearly independent over $\mathbb{R}$.

(b) Define T: $\mathbb{R}^3 \longrightarrow \mathbb{R}^2$ by $T(a_1,a_2,a_3) = (a_1-a_2,2a_3)$. Prove that T is a linear transformation and find a basis of both N(T) and R(T). Also verify the dimension theorem.

6. (a) For the vector space, Let $V = \left\{\begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix} : a, b \in \mathbb{R}\right\}$.

Find a basis of V over $\mathbb{R}$.

(b) Let T: $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear transformation such that T(1,2)=(2,3) and T(0,1)=(1,1). Find T(a,b) for any (a,b) ∈ $\mathbb{R}^2$.