| Unique Paper Code | $: \mathbf{3 2 3 5 5 3 4 5}$ |
| :--- | :--- |
| Name of Paper | $:$ Linear Programming and Game Theory (NC) |
| Name of Course | $:$ :III |
| Semester | $: \mathbf{3}$ hours |
| Duration | $: \mathbf{7 5}$ |

## Attempt any four questions. All questions carry equal marks.

1. Find all the basic feasible solutions of the following equations

$$
\begin{aligned}
2 x_{1}+3 x_{2}+4 x_{3}+x_{4} & =6 \\
x_{1}+x_{2}+7 x_{3}+x_{4} & =2 .
\end{aligned}
$$

Use Simplex method to find the inverse of the matrix $\left[\begin{array}{ll}1 & 4 \\ 2 & 5\end{array}\right]$.
2. Solve the following linear programming problem using Big-M method

$$
\begin{aligned}
& \text { Maximize } z=2 x_{1}+4 x_{2}+4 x_{3}-3 x_{4} \\
& \text { Subject to } \quad 2 x_{1}+x_{2}+x_{3}=4 \\
& x_{1}+4 x_{2}+\quad 3 x_{4}=6 \\
& x_{1}, x_{2}, x_{3}, x_{4} \geq 0 .
\end{aligned}
$$

3. Let $x_{0}$ be feasible solution of primal linear programming problem-lpp and $w_{0}$ be a feasible solution of its dual, if the objective values of primal and dual lpp are equal show that $x_{0}$ and $w_{0}$ are the optimum solutions to the primal and dual lpp respectively.
Obtain the dual linear program of the following primal linear program:

$$
\begin{aligned}
\text { Minimize } z= & -2 x_{1}+3 x_{2}+5 x_{3} \\
\text { Subject to } \quad & -2 x_{1}+x_{2}+3 x_{3}+x_{4} \geq 5 \\
& 2 x_{1}+. \quad x_{3}+x_{4}=6
\end{aligned} \quad \begin{aligned}
& \\
& x_{1} \leq 0, x_{2}, \\
& x_{3} \geq 0 ; x_{4} \text { Unrestricted in sign. }
\end{aligned}
$$

4. Given a transportation problem:

| Destinations |  | P | Q | R | S | Availability |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Origin | A | 11 | 9 | 7 | 10 | 120 |
|  | B | 5 | 11 | 9 | 6 | 115 |
|  | C | 4 | 7 | 8 | 6 | 210 |
|  | D | 3 | 12 | 4 | 5 | 105 |
|  | Requirements |  |  |  | 95 | 115 | 140 |
| 2 |  |  |  |  |  |  |

Compare the initial basic feasible solutions for the given transportation problem using
(i) Least Cost Method
(ii) North-West Corner Method
(iii) Vogel's Approximation Method-VAM,

Also find the optimal solution of the transportation problem using VAM for initial basic feasible solution.
5. Solve the following cost minimizing assignment problem:

|  | I | II | III | IV | V | VI |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| A | 7 | 3 | 3 | 7 | 6 | 2 |
| B | 5 | 7 | 7 | 5 | 5 | 7 |
| C | 3 | 7 | 9 | 3 | 1 | 6 |
| D | 6 | 7 | 8 | 6 | 9 | 4 |
| E | 5 | 3 | 7 | 5 | 6 | 3 |
| F | 8 | 4 | 8 | 7 | 2 | 2 |

6. For the following payoff matrix $\left[\begin{array}{cc}2 & 6 \\ -2 & x\end{array}\right]$ of a game, show that the game has a saddle point whatever $x$ may be. Find the value of this game and determine the saddle point.

Solve the following game graphically:

| Player B |  |  |
| :--- | :--- | :--- |
| Player A | 2 | 1 |
|  | 1 | 0 |
|  | 0 | 3 |
|  | -2 | 2 |

