Sr. No. of Question Paper:

Unique Paper Code: 12277504 - OC

Name of the Paper: Topics in Microeconomics - I

Name of the Course: CBCS DSE

Semester: V

Maximum Marks: 75

Instructions for Candidates

There are *six* questions in all. Answer any *four* questions. All questions carry equal marks.

1. (a) Consider the payoff matrix of a two player game given below where $a_i > 0, b_i \in \mathbb{R}$. Player 1's actions are given in the rows and player 2's in the columns.

	H	T
H	$(a_1 + b_1), (-a_2 + b_2)$	$(-a_1 + b_1), (a_2 + b_2)$
T	$(-a_1 + b_1), (a_2 + b_2)$	$(a_1 + b_1), (-a_2 + b_2)$

(i) Verify that the game has no pure strategy Nash equilibrium.
(ii) Find the mixed strategy Nash equilibrium/equilibria of this game.

(b) Two candidates, D and R, are contesting in an election for the post of a mayor in a town with n voters. A total of $0 < d < n$ residents support candidate D and the remaining $r = n - d$, support candidate R. The value for each resident of having his candidate win is 4, for having him tie is 2 and for having him lose is 0. Going to vote costs each resident 1. The strategic game has the voters as the players. Each of them will have to decide whether to vote or not vote for the candidate whom they support.
(i) Let $n = 2$ and $d = 1$. Write down this game using a payoff matrix and solve for the pure strategy Nash equilibrium/equilibria.
(ii) Let $n > 2$ be an even number and let $d = r = \frac{n}{2}$. Find the Nash equilibrium/equilibria.
(iii) Assume now that the cost of voting is equal to 3. How do your answers to parts (i) and (ii) change?

2. (a) Consider a sealed-bid auction in which two bidders simultaneously submit bids for a single prize worth *10* to each of them. The bids must be non-negative integers less than or equal to *10*. If the bids are unequal, the player who submits the higher bid gets the prize. If the bids are equal, neither player gets anything. Each player must pay the amount that he has bid, whether or not he wins. Players maximize their expected net gain. Does this game have any Nash equilibrium in pure strategies? Find a symmetric mixed strategy Nash equilibrium in which every bid less than *10* has a positive probability.

(b) Two players take turns removing stones from a pile of *4* stones. Player *1* moves first. Each player has to remove one or two stones on each of his turns. The player who removes the last stone is the winner and gets Rs *100* from the other player. Show that player *1* is the winner in any subgame perfect equilibrium of this game. What would happen if the pile had *6* stones?

3. (a) Consider the Bertrand duopoly game where the cost function of each firm is $C_i(q_i) = c_i q_i$, where $0 < c_1 < c_2$. The market demand function is $D(P) = a - P$ for $P < a$, zero otherwise. Assume that $c_2 < (a. - c_1)/2$. Firms simultaneously choose the prices they charge. Prices are restricted to be non-negative integers. The firm that charges a lower price captures the entire market. Further

suppose that when both firms charge the same price, the market is equally split amongst the firms. Find all the pure strategy Nash equilibria of this game.

(b) Two candidates *X* and *Y* contest an election. Of the *100* citizens, *m* support candidate *X* and *n* *(=100 – m)* support candidate *Y*. Each citizen has two strategies. He can either vote for his favourite candidate or he can abstain from voting. The opportunity cost of voting is *c*, where $0 < c < 5$. The utility of a citizen who abstains is *10* if his favourite candidate wins outright, *5* in case of a tie and zero in case he loses. In case the citizen votes, *c* is subtracted from his utility in all the three contingencies.

(i) For $m = 50$, find the set of pure strategy Nash equilibria of this game.

(ii) What is the set of pure strategy Nash equilibria for $m = 60$?

4. (a) Player *1* is a police officer who must decide whether to patrol the streets or to hang out at the coffee shop. His payoff from hanging out at the coffee shop is *10*, while his payoff from patrolling the streets depends on whether he catches a robber, who is player 2. If the robber prowls the streets then the police officer will catch him and obtain a payoff of *20*. If the robber stays in his hideaway then the officer's payoff is *0*. The robber must choose between staying hidden or prowling the streets. If he stays hidden then his payoff is *0*, while if he prowls the streets his payoff is -10 if the officer is patrolling the streets and *10* if the officer is at the coffee shop.

(i) Write down the matrix form of this game.

(ii) Draw the best-response function/correspondence of each player.

(iii) Find the Nash equilibrium of this game.

(b) An object that two people each value at v (a positive integer) is sold in an auction. In the auction, the people alternately have the opportunity to bid; a bid must be a positive integer greater than the previous bid. On her turn, a player may pass rather than bid, in which case the game ends and the other player receives the object; both players pay their last bid (if any). Each person's wealth is w, which exceeds v; neither player may bid more than her wealth. For $v = 2$ and $w = 3$ model this auction as an extensive game with perfect information and find its subgame perfect equilibrium/equilibria.

5. Consider three firms in an industry such that each firm $i = 1,2,3$ produces quantity q_i. Firms produce their output sequentially with firm 1 choosing how much to produce first, followed by firm 2 and then firm 3. The market price is given by:

$$p = 120 - Q$$

Where $Q = q_1 + q_2 + q_3$. The marginal cost is assumed to be a constant and same for all firms and equal to c.

(a) Set up this description as an extensive game with perfect information and determine the subgame perfect equilibrium of this game. Find the equilibrium price p^* as well as the equilibrium profits $\pi_1^*, \pi_2^*, \pi_3^*$. All these equilibrium values should be expressed in terms of only c.

(b) p^*, the equilibrium price should be greater than c for firms to make positive profits. What condition should c satisfy for this to be the case? Assume that c satisfies this condition. Which firm produces the highest output in the subgame perfect equilibrium? Which firm produces the lowest? Which firm receives the highest profits in the subgame perfect equilibrium and which firm the lowest?

6. Consider the demand function

$$q_i = 2 - 2p_i + p_j$$

$i = 1,2$, $i \neq j, j$ is the other firm, q_i is quantity and p_i the price charged by firm i.

Assume that the firms move sequentially and choose their prices with firm 1 moving first followed by firm 2. Both firms want to maximize their profits and both firms' costs are zero.

(a) Set this up as an extensive game with perfect information.

(b) Solve for the subgame perfect equilibria and also find the profits of both the firms.

(c) How is the subgame perfect outcome above different from the outcome that would have resulted had the firms moved simultaneously instead of sequentially? Does the firm moving first enjoy a higher level of profits than the firm moving second in the sequential move game? Compare these profit levels to the profit levels of both firms when they take the decisions simultaneously.