Name of the Course	**: CBCS B.Sc. (Mathematical Sciences) / B.Sc. (Physical Sciences)/ B.Sc. (Life Sciences)/Applied Sciences**
Unique Paper Code	**: 42357501**
Name of the Paper	**: DSC-Differential Equations**
Semester	: **V**
Duration	: **3 Hours**
Maximum Marks	: **75**

*Attempt any **four** questions. All questions carry equal marks.*

1. Find an integrating factor and solve the differential equation

$$(y^2x + y^2 + y)dx + (2xy + 1)dy = 0$$

Solve the differential equation

$$3\,x\,(1 - x^2)y^2\frac{dy}{dx} + (2x^2 - 1)y^3 = x^3$$

Also, solve the differential equation

$$(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0$$

2. Solve the differential equation

$$\frac{d^2y}{dx^2} - y = x^2\cos x$$

Also, solve the following initial value problem

$$x^2\frac{d^2y}{dx^2} - 2y = 4x - 8, \qquad y(1) = 4, y'(1) = -1$$

3. Given that $y = e^{2x}$ is a solution of

$$(2x + 1)\frac{d^2y}{dx^2} - 4(x + 1)\frac{dy}{dx} + 4y = 0,$$

Find the linearly independent solution by reducing the order and also write the general solution.

Use the method of variation of parameters to solve the equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \frac{e^{-3x}}{x^3}$$

4. Find a family of oblique trajectories that intersect the family of parabolas $y^2 = cx$ at angle 60^o.

   Also, solve the system of linear differential equations

$$2\frac{dx}{dt} + 4\frac{dy}{dt} + x - y = 3e^t$$

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + 2y = e^t$$

5. Find the general solution of the equation

$$(y - xu)\frac{\partial u}{\partial x} + (x + yu)\frac{\partial u}{\partial y} = x^2 - y^2$$

   Apply the method of separation of variables to solve the equation:

$$\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + u = 0, \qquad with\, u(x,0) = 4e^{-3x}$$

6. Find the partial differential equation arising from the surfaces

$$z = f(x^2 + y^2)$$

   Also, reduce the equation

$$\frac{\partial^2 u}{\partial x^2} + 2x\frac{\partial^2 u}{\partial x \partial y} + x^2\frac{\partial^2 u}{\partial y^2} = 0$$

   to canonical form.