Set-1

Name of the Course	**:** CBCS B.Sc. (Hons.) Mathematics
Unique Paper Code	**:** 32357505
Name of the Paper	**:** DSE-II Discrete Mathematics
Semester	**:** V
Duration	**:** 3 Hours
Maximum Marks	**:** 75

Instructions for Candidates

Attempt any four questions. All questions carry equal marks.

Q1. Prove or disprove the statement "Every lattice ordered set is a lattice as an algebra". Give an example of an infinite lattice with and without least and greatest elements. Draw the Hasse diagram of $\mathbf{M}_2 \oplus \mathbf{M}_3$.

Q2. Let $(L, \leq)$ be a lattice with greatest element 1 and let $a \vee b = 1 = (a \wedge b) \vee c, \forall\, a, b, c \in L$. Then verify that $a \vee (b \wedge c) = 1$. Also, verify that $b \vee (c \wedge a) = a \vee (b \wedge c)$ and $c \vee (a \wedge b) = a \vee (b \wedge c)$.
Let L be the set of divisors of 30 equipped with divisibility order. Find complement of each member of L.

Q3. Write the Disjunctive normal form of Boolean Polynomial

$$p = x\,(y + z)' + (x\,y + z)\,x'.$$

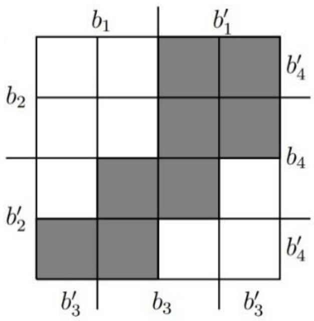
Let A be the set of all atoms in a finite Boolean Algebra B. Define a function

$$h : B \to P(A)$$ and verify that it is a lattice homomorphism.

Q4. Use the Quine-McCluskey method to find the minimal form of

$$x'y'z' + xy'z + xy'z' + xyz' + xyz + x'y'z + x'yz.$$

A circuit p is given by $p = (x_1' + x_2 + x_3)(x_1' + x_2' + x_3)$. Simplify p and find the symbolic representation of the reduced expression of the circuit.
Find simple function for the following Karnaugh diagram:

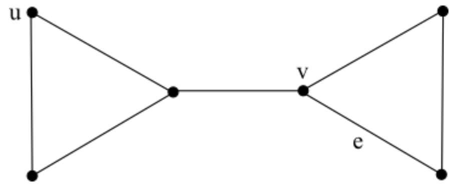


Q5. A graph has 50 edges, four vertices of degree 2, six vertices of degree 5, eight of degree 4 and the rest of degree 6. How many vertices does G have?

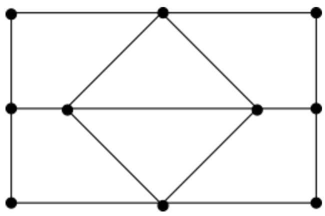
Does there exist a graph with degree sequence 6,6,5,4,3,2,1. Either draw a graph or explain why no such graph can exist.

Find out for what values of *n*, the complete graph of order *n* ($n \geq 1$) and cycle of length *n* ($n \geq 3$) are bipartite graphs?

For the graph shown below, draw pictures of the subgraphs $G - \{e\}$, $G - \{v\}$ and $G - \{u\}$.



Consider the following graph.

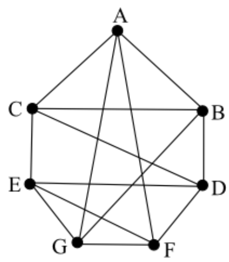


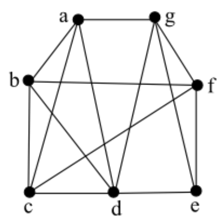
(i) Is it Eulerian?

(ii) Is there an Eulerian trail?

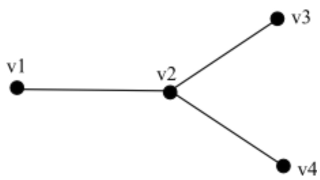
Explain your answer.

Q6. For the graphs shown below, either exhibit an isomorphism between vertex sets or explain why the graphs are not isomorphic.





Let A be the adjacency matrix of the graph shown below.



Without actually computing A^2, A^3 and A^4 by matrix multiplication determine the values of the following entries:

(i) (1,3) entry of A^2	(ii) (3,4) entry of A^2
(iii) (1,3) entry of A^3	(iv) (2,4) entry of A^3
(v) (1,2) entry of A^3	(vi) (2,3) entry of A^4

Apply the improved version of Dijkstra's Algorithm to find the shortest path from A to D in the graph shown. Write steps.