Set - D

S. No. of Question Paper :

Unique Paper Code : **32227519**

Name of the Paper : **Linear Algebra and Tensor Analysis (DSE)**

Name of the Course : **B.Sc. (Hons) Physics CBCS**

Semester : **V**

Duration: **3 + 1 hours** Maximum marks: **75**

Attempt any four questions. All questions carry equal marks.

1. (a) Determine whether the identity element exist or not for the binary operation $*$ defined as:

$$a * b = a^b$$

(b) Check whether the subset of R^3 defined as

$$\{[x_1, x_{2,} x_3] \mid x_1 x_2 = x_3\}$$

forms a subspace or not.

(c) Consider the vector space $\boldsymbol{P}_3(t)$ of polynomials of degree ≤ 3. Find the coordinate vector $[v]_S$ of $v = 3t^3 - 4t^2 + 2t - 5$ relative to the basis S where

$$S = \{(t-1)^3, (t-1)^2, (t-1), 1\}$$

is a basis of $\boldsymbol{P}_3(t)$. (4.75 + 5 + 9)

2. (a) Find the dimension of, and a basis for, the solution space of the following system:

$$x_1 + 2x_2 - 4x_3 + 3x_4 - x_5 = 0$$
$$x_1 + 2x_2 - 2x_3 + 2x_4 + x_5 = 0$$
$$2x_1 + 4x_2 - 2x_3 + 3x_4 + 4x_5 = 0$$

(b) A linear transformation T on R^3 is represented relative to the ε- basis by the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & -2 \\ -1 & 2 & 3 \end{bmatrix}$$

Let $\alpha_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\alpha_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\alpha_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

(i) Determine the matrix **B** that represents T relative to the α-basis by computing each $T\alpha_i$ and expressing it in terms of the α-basis.

(ii) Find the transition matrix **P** from the ε-basis to the α-basis and show that $P^{-1}AP = B$. (6.75 + 12)

3. (a) Write the following matrix A as a product of elementary matrices

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$$

(b) Evaluate 4^A, where A is

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

(8.75 +10)

4. (a) Consider the vectors $\{(1, \;\; 2, \;\; -1), (2, \;\; 0, \;\; 2), (1, \;\; -2, \;\; 3)\}$ in $\boldsymbol{R}^3$. Using Gram Schmidt orthogonalization process construct an orthonormal basis for this space.

(b) Find the eigen values of the following matrix

$$A = \begin{bmatrix} p & q & q \\ q & p & q \\ q & q & p \end{bmatrix}$$

(c) Given a vector $\vec{A} = (u, u + v, u + v + w)$. Find the elements of the second order skew-symmetric tensor associated with it. (6 + 6.75 + 6)

5. (a) If A_q is a vector, show that

$$F_{pq} = \frac{\partial A_p}{\partial x_k} + \frac{\partial A_k}{\partial x_p}$$

is a second order cartesian tensor.

(b) Force $\vec{F} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$ acts on a cuboid $0 \leq x \leq 6, 0 \leq y \leq 4$ and $0 \leq z \leq 2$. Find stress tensor acting on the cuboid. Also find the principal stresses. (6 + 12.75)

6. (a) Using tensors, prove that the divergence of curl of a vector is zero.

(b) In a three-dimensional cartesian coordinate system (x, y, z), let $\vec{B} = B_0\hat{x}$, where B_0 is a constant. Find the contravariant and covariant components of this constant vector $\vec{B}$ in spherical polar coordinates. (6.75 + 12)