SET A

Name of Course : **B.Sc. Hons. Physics-CBCS_NC_Core**

Semester : **I**

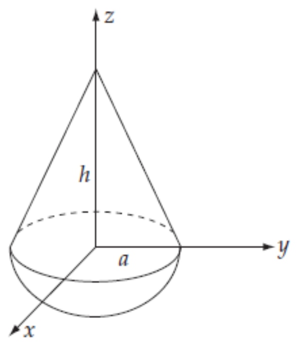
Name of Paper : **Mechanics**

Unique Paper Code : **32221102**

Duration : **3 Hours** Maximum Marks: **75**

Answer any **four** of the six questions. Each question carries equal marks.

1. **(a)** Find the center of mass of a uniformly solid cone of base diameter $2a$ and height h and a solid hemisphere of radius a where the two bases are touching.



(b) A rocket leaves Earth's surface under gravity, typically in a vertical direction and returns to Earth. The exhaust velocity is u, and the constant fuel burn rate is α. The initial mass is m_o and the mass at fuel burnout is m_f. Show that the altitude of the rocket at fuel burnout is given by

$$H = -\frac{g(m_o - m_f)^2}{2\alpha^2} + \frac{u}{\alpha}\left[m_f \ln\left(\frac{m_f}{m_o}\right) + m_o - m_f\right].$$

(c) A child slides a block of mass 2 kg along a slick kitchen floor. If the initial speed is 4 m/s and the block hits a spring with spring constant 6 N/m, what is the maximum compression of the spring? What is the result if the block slides across 2 m of a rough floor that has $\mu_k = 0.2$?

(6.75+7+5)

2. **(a)** Check if the following forces are conservative. If conservative, find the potential energy $U(\vec{r})$.

$$F_x = ayz + bx + c, \quad F_y = axz + bz, \quad F_z = axy + by.$$

(b) A particle of mass m moving in one dimension has potential energy

$$U(x) = U_o\left[2\left(\frac{x}{a}\right)^2 - \left(\frac{x}{a}\right)^2\right]$$ where U_o and a are positive constants.

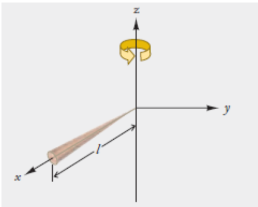
i. Sketch $U(x)$ on an energy diagram and locate the position of stable and unstable equilibrium.
ii. What is the angular frequency ω of oscillations about the point of stable equilibrium?
iii. What is the minimum speed the particle must have at the origin to escape to infinity?

(c) A particle of mass m with initial velocity u_o collides with an unknown particle at rest. After collision, the mass m scattered through an angle of 45^o with its original line of motion. The unknown particle moves with speed $-u_o/5$ in the centre of mass frame.

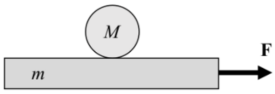
i. Find the final velocities of each particle in the Lab frame.
ii. Find the scattering angle of the unknown particle in the C −frame.

(5+8+5.75)

3. (a) Show that the moment of inertia of a long, very thin cone about an axis through the apex and perpendicular to the centerline is $\frac{3}{5}Ml^2$, where M is the mass and l is the height of the cone.

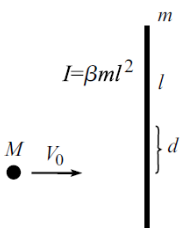


(b) A uniform cylinder of mass M and radius R is at rest on a uniform block of mass m, which in turn rests on a horizontal, frictionless table. If a horizontal force F is applied to the block, the block accelerates and the cylinder rolls without slipping.



i. Find the acceleration of the block.
ii. Find the angular acceleration of the cylinder.
iii. If the force F acts over a distance d, calculate the kinetic energy of the block.

(c) A ball of mass M collides with a stick with moment of inertia $I = \beta ml^2$ (relative to its center, which is its center of mass). The ball is initially traveling at speed V_o perpendicular to the stick. The ball strikes the stick at a distance d from the center. The collision is elastic. Find the angular speed of the stick after collision.



(7+6+5.75)

4. (a) Show that the steady state amplitude of a damped oscillator driven by an external force $F_o e^{i\omega t}$ is given by the expression

$$A = \frac{F_o}{m[(\omega_o^2 - \omega^2)^2 + \gamma^2\omega^2]^{1/2}}$$

where m is the mass of the system, ω_o is the natural frequency of the oscillator, ω is the driving frequency, and γ is the damping constant. Discuss the amplitude resonance.

(b) The density of a sphere is given by $\rho(r) = \frac{k}{r}$ where k is a constant. The sphere has a radius of $5.0\ m$ and a mass of $1011\ kg$.

i. Determine the constant k.
ii. Find the gravitational field for the region $r < 5.0\ m$.

(c) A particle of mass m moves along a trajectory given by $x = x_o \cos \omega_1 t$, $y = y_o \sin \omega_2 t$.

i. Find the x and y components of the force. Under what condition is the force a central force?
ii. Find the potential energy as a function of x and y.
iii. Determine the kinetic energy of the particle. Show that the total energy of the particle is conserved.

(8+4+6.75)

5. **(a)** Show that the expression for the acceleration in the fixed coordinate system in terms of the position, velocity, and acceleration in the rotating coordinate system is given by

$$\vec{a} = \vec{a}' + \dot{\vec{\omega}} \times \vec{r}' + 2\vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}').$$

Give the physical interpretation of the terms involved in the equation.

(b) A rocket that has a proper length of $1000\ m$ moves away from a space station and in the positive $x -$ direction at $0.60\ c$ relative to an observer on the station. An astronaut stands at the rear of the rocket and fires a dart toward the front of the rocket at $0.80\ c$ relative to the rocket. How long does it take to reach the front of the rocket as measured in the frame of

i. the rocket.
ii. the space station.

(c) Two light sources A and B situated 10 meters apart flash light signals at an interval of one nanosecond. At what time interval will an observer traveling at a speed of $0.9\ c$ along the direction AB see the two events? Which source A or B that he will find the flash first?

(7+6+5.75)

6. **(a)** Derive relativistic transformation equations for momentum and energy. Show that $p^2 - \frac{E^2}{c^2}$ is Lorentz invariant, where the symbols have their usual meaning.

(b) A particle having rest mass m_0 and kinetic energy xm_oc^2, where x is some number, strikes an identical particle at rest and sticks to it. What is the rest mass of the resultant particle?

(c) High-energy neutrino beams at Fermi laboratory are made by first forming a monoenergetic π^+ beam and then allowing the pions to decay by $\pi^+ \rightarrow \mu^+ + \nu$. Note that the mass of the pion is $140\ MeV/c^2$ and the mass of the muon is $106\ MeV/c^2$. Find the energy of the decay neutrino in the rest frame of the π^+.

(7+6.75+5)