Name of the Department	:	Physics
Name of the Course	:	B.Sc. (H) Physics – CBCS – NC - Core
Semester	:	I
Name of the Paper	:	Mathematical Physics-I
Unique Paper Code	:	32221101
Question Paper Set Number	:	A
Maximum Marks	:	75

Instruction for Candidates

Attempt **FOUR** questions in all.
All questions carry equal marks.

1. Solve the following differential equations.

(a) $x^2\frac{dy}{dx} + 2xy = 5y^3$

(b) $\frac{dy}{dx} = \frac{x-y-1}{x+y+3}$

(c) Determine the most general function $N(x, y)$ such that the following equation is exact and hence solve it.

$$(x^3 + xy^2)dx + N(x, y)dy = 0$$

2. Solve the following differential equations.

(a) $(D^2 + 4)y = x \sin 2x$

(b) $(2D^2 + 4D + 7)y = x^2$

(c) $(D^2 + 3D + 2)y = 4e^x$ (Use the method of variation of parameters)

3. **(a)** Show that the shortest distance between two points in a plane is a straight line.

(b) A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 6x(1-x), & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Show that it is a probability density function. Find its mean and variance.

4. **(a)** Find the directional derivative of $\varphi = x^2 + xy + y^2$ at (1,-1) in direction towards the origin.

(b) Show that $f(r)\,\vec{r}$ is irrotational, where $f(r)$ is differentiable.

(c) Compute the integral given below for rectangle R such that, $1 \le x \le 3, 1 \le y \le 2$

$$\iint_R \frac{xy}{x^2 + y^2} dA$$

5. **(a)** Verify the divergence theorem for $\vec{A} = 2x^2y\hat{\imath} - y^2\hat{\jmath} + 4xz^2\hat{k}$ taken over the region in the first octant bounded by $x^2 + y^2 = 9$ and $x = 2$.

(b) Let $\vec{A} = (-4x - 3y + 5z)\hat{\imath} + (-3x + 3y + 5z)\hat{\jmath} + (4x + 5y + 3z)\hat{k}$. Show that $\vec{A}$ can be expressed as the gradient of a scalar function.

6. **(a)** Verify Green's theorem for the field,

$$\vec{F}(x, y) = (x - y)\hat{\imath} + x\hat{\jmath}$$

It is given that region R is bounded by unit circle C:

$\vec{r}(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath}\,; 0 \leq t \leq 2\pi$

(b) Find components of a vector $\vec{A} = 2y\hat{\imath} - z\hat{\jmath} + 3x\hat{k}$ in cylindrical coordinate system