DSE: Advanced Mathematical Physics - I (32227502)
Credit : 06 (Theory-04, Practical-02)
Theory : 60 Hours
Practical : 60 Hours

Course Objective

The course is intended to impart the concept of generalized mathematical constructs in terms of Algebraic Structures (mainly Vector Spaces) and Tensors to have in-depth analysis of our physical system.

Course Learning Outcomes

At the end of this course, students will be able to

- Understand algebraic structures in n-dimension and basic properties of the linear vector spaces.
- Represent Linear Transformations as matrices and understand basic properties of matrices.
- Apply vector spaces and matrices in the quantum world.
- Learn basic properties of Cartesian and general tensors with physical examples such as moment of inertia tensor, energy momentum tensor, stress tensor, strain tensor etc.
- Learn how to express the mathematical equations for the Laws of Physics in their co-variant forms.
- In the laboratory course, the students are expected to solve the problems using the Scilab/C++/Python computer language: Eigenvalues and Eigenvectors of given matrix, determination of wave functions for stationary states as eigenfunctions, eigen energy values of Hermitian differential operators, Lagrangian formulation in classical dynamics etc.

Unit 1

Linear Vector Spaces Abstract Systems: Binary Operations and Relations. Introduction to Groups and Fields.

Vector Spaces and Subspaces. Linear Independence and Dependence of Vectors. Basis and Dimensions of a Vector Space. Change of basis. Homomorphism and Isomorphism of Vector Spaces. Linear Transformations. Algebra of Linear Transformations. Non-singular Transformations. Representation of Linear Transformations by Matrices.

(12 Lectures)

Unit 2

Matrices, Addition and Multiplication of Matrices: Null Matrices. Diagonal, Scalar and Unit Matrices. Upper- Triangular and Lower-Triangular Matrices. Transpose of a Matrix. Symmetric and Skew-Symmetric Matrices. Conjugate of a Matrix. Hermitian and Skew-Hermitian Matrices. Singular and Non-Singular matrices. Orthogonal and Unitary Matrices. Similar Matrices. Trace of a Matrix. Inner Product.

(8 Lectures)

Unit 3

Eigen-values and Eigenvectors: Finding Eigen – values and Eigen vectors of a Matrix. Diagonalization of Matrices. Properties of Eigen-values and Eigen Vectors of Orthogonal, Hermitian and Unitary Matrices. Cayley-Hamiliton Theorem (Statement only). Finding inverse of a matrix using Cayley-Hamiltion Theorem. Use of Matrices in Solving ordinary second order differential equations and Coupled Linear Ordinary Differential Equations of first order. Functions of a Matrix.

(10 Lectures)

Unit 4

Cartesian Tensors: Transformation of Co-ordinates and fundamentals of Tensors. Einstein's Summation Convention. Relation between Direction Cosines. Algebra of Tensors: Sum, Difference and Product of Two Tensors. Contraction. Quotient Law of Tensors. Symmetric and Anti-symmetric Tensors. Invariant Tensors: Kronecker and Alternating Tensors. Association of Anti-symmetric Tensor of Order Two and Vectors.

(8 lectures)

Unit 5

Applications of Cartesian Tensors: Vector Calculus using Cartesian Tensors: Scalar and Vector Products of 2, 3, 4 vectors. Gradient, Divergence and Curl of Tensor Fields. Tensor notation of Laplacian operator. Proof of Vector Identities involving scalar and vector products and vector identities involving Del operator using Tensor notation. Isotropic Tensors (Definition only). Tensorial Character of Physical Quantities. Moment of Inertia Tensor. Stress and Strain Tensors: Symmetric Nature. Elasticity Tensor. Generalized Hooke's Law.

(12 lectures)

Unit 6

General Tensors: Transformation of Co-ordinates. Contravariant & Covariant Vectors. Contravariant, Covariant and Mixed Tensors. Kronecker Delta and Permutation Tensors. Algebra of Tensors. Sum, Difference & Product of Two Tensors. Contraction. Quotient Law of Tensors. Symmetric and Anti- symmetric Tensors. Metric Tensor in cartesian, cylindrical, spherical coordinates.

(10 Lectures)

Practical: 60 Hours

PRACTICAL- DSE LAB: Advanced Mathematical Physics-I

Scilab/C^{++}/Python based simulations experiments based on Mathematical Physics problems like (at least 06 experiments)

1. Linear algebra: Power and Inverse Power methods for finding largest and smallest Eigenvalue and eigenvectors of matrices. QR method e.g.

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ 3 & 1 & 4 \end{pmatrix}; \begin{pmatrix} 1 & -i & 3+4i \\ +i & 2 & 4 \\ 3-4i & 4 & 3 \end{pmatrix}; \begin{pmatrix} 2 & -i & 2i \\ +i & 4 & 3 \\ -2i & 3 & 5 \end{pmatrix}$$

2. Orthogonal polynomials as eigenfunctions of Hermitian differential operators.

3. Determination of the principal axes of moment of inertia through diagonalization (Matrix can be generated for a given distribution of discrete masses).
4. Study of geodesics in Euclidean and other spaces (surface of a sphere, etc): Using variational principal to find the shortest curve between two points. Suggested Physics problem: problem of refraction.
5. Application to solve differential equations for a bound system – Eigen value problem.
6. Application to computer graphics:
   Write operators for shear, strain, two dimensional rotational problems, Reflection, Translation etc. Plot old and new coordinates.
7. Lagrangian formulation in classical mechanics with constraints.
8. Vector space of wave functions in Quantum Mechanics: Position and Momentum differential operators and their commutator, wave functions for stationary states as eigenfunction

----

Note: Students opting for Linear algebra and Tensor analysis as one option in DSE cannot opt Advanced mathematical physics-I course as second option.

References for Theory:

Essential Readings:

1. Mathematical Tools for Physics, James Nearing, 2010, Dover Publications
2. Theory and Problems of Linear Algebra, Seymour Lipschutz, 1987, McGraco-Hill Inc.
3. Theory and Problems of Vector Analysis and an introduction to Tensor Analysis, Murray R. Spiegel, 1974, McGraw Hill, Inc.
4. Introduction to Matrices & Linear Transformations, D.T.Finkbeiner,1978, Dover Pub.
5. Matrices and tensors in Physics: A.W. Joshi, New Age International Pvt. Ltd (2017).

Additional Readings:

1. Mathematical Methods for Physicists, G.B. Arfken, H.J. Weber and F.E.Harris,1970, Elsevier.
2. Elementary Linear Algebra, Applications Version, Howard Anton and Chris Rorres, Wiley Student edition.
3. Mathematics for Physicists, Susan M. Lea, 2004, Thomson Brooks/Cole
4. Introduction to Vectors and Tensors, Ray M Bowen, C -C Wang, Dover Publications (2009)
5. An Introduction to Linear Algebra and Tensors, M A Akivis, V V Goldberg, Richard and Silverman, Dover Publications (2012)
6. Vector Analysis and Cartesian Tensors, D.E. Bourne and P.C. Kendall, CRC Press (1992).
7. Cartesian Tensors, Harold Jeffreys, Cambridge University Press (1931).

References for Laboratory Work:

1. Scilab by example: M. Affouf, 2012, ISBN: 978-1479203444
2. Learning Scientific Programming with Python, Christian Hill, Cambridge University Press (2016)
3. Computational Problems for Physics: With Guided Solutions Using Python, Rubin H. Landau, Manuel José Páez, CRC Press (2018).
4. Numerical Recipes in C^{++}: The Art of Scientific Computing, W.H. Press et.al., 2nd Edn., Cambridge University Press (2013).
5. Elementary Numerical Analysis, K.E. Atkinson, 3rd Edn., 2007, Wiley India Edition.

DSE: Nuclear and Particle Physics (32227504)
Credit : 06 (Theory-05, Tutorial-01)
Theory : 75 Hours
Tutorial : 15 Hours

Course Objective

The objective of the course is to impart the understanding of the sub atomic particles and their properties. It will emphasize to gain knowledge about the different nuclear techniques and their applications in different branches Physics and societal application. The course will focus on the developments of problem based skills.

Course Learning Outcomes

- To be able to understand the basic properties of nuclei as well as knowledge of experimental determination of the same, the concept of binding energy, its various dependent parameters, N-Z curves and their significance
- To appreciate the formulations and contrasts between different nuclear models such as Liquid drop model, Fermi gas model and Shell Model and evidences in support.
- Knowledge of radioactivity and decay laws. A detailed analysis, comparison and energy kinematics of alpha, beta and gamma decays.
- Familiarization with different types of nuclear reactions, Q- values, compound and direct reactions.
- To know about energy losses due to ionizing radiations, energy losses of electrons, gamma ray interactions through matter and neutron interaction with matter. Through the section on accelerators students will acquire knowledge about Accelerator facilities in India along with a comparative study of a range of detectors and accelerators which are building blocks of modern day science.