

5. Digital Design, Morris Mano, 5th Ed. Pearson.

Additional Readings :

1. Digital Electronics G K Kharate ,2010, Oxford University Press
2. Logic circuit design, Shimon P. Vingron, 2012, Springer
3. Digital Electronics, Subrata Ghoshal, 2012, Cengage Learning. Digital Electronics, S.K. Mandal, 2010, 1st edition, McGraw Hill

References for Laboratory Work :

1. Modern Digital Electronics, R.P. Jain, 4th Edition, 2010, Tata McGraw Hill
2. Basic Electronics: A text lab manual, P.B.Zbar, A.P.Malvino, M.A.Miller, 1994, McGraw Hill.
3. Microprocessor 8085: Architecture, Programming and interfacing, A.Wadhwa,2010, PHI Learning

CC-VIII: Mathematical Physics III (32221401)

Credit : 06 (Theory-04, Practical-02)

Theory : 60 Hours

Practical : 60 Hours

Course Objective

The emphasis of the course is on applications in solving problems of interest to physicists. Students will be examined on the basis of problems, seen and unseen. The course will develop understanding of the basic concepts underlying complex analysis and complex integration and enable student to use Fourier and Laplace Transform to solve real world problems.

Course Learning Outcomes

After completing this course, student will be able to

- Determine continuity, differentiability and analyticity of a complex function, find the derivative of a function and understand the properties of elementary complex functions.
- Work with multi-valued functions (logarithmic, complex power, inverse trigonometric function) and determine branches of these functions
- Evaluate a contour integral using parametrization, fundamental theorem of calculus and Cauchy's integral formula.

- Find the Taylor series of a function and determine its radius of convergence.
- Determine the Laurent series expansion of a function in different regions, find the residues and use the residue theory to evaluate a contour integral and real integral.
- Understand the properties of Fourier and Laplace transforms and use these to solve boundary value problems.
- In the laboratory course, the students will learn the basics of the Scilab software/Python interpreter and apply appropriate numerical method to solve selected physics problems both using user defined and inbuilt functions from Scilab/Python.

Unit 1

Complex Analysis

Complex Analysis: Brief Revision of Complex Numbers and their Graphical Representation. Euler's formula, De-Moivre's theorem, Roots of Complex Numbers. Functions of Complex Variables. Analyticity and Cauchy-Riemann Equations. Examples of analytic functions. Singularities: poles, removable singularity, essential singularity, branch points, branch cut. Integration of a function of a complex variable. Cauchy-Goursat Theorem, Cauchy's Inequality. Cauchy's Integral formula. Simply and multiply connected region. Laurent and Taylor's expansion. Residues and Residue Theorem. Application of Contour Integration in solving Definite Integrals.

(30 Lectures)

Unit 2

Integrals Transforms

Fourier Transforms: Fourier Integral theorem (Statement only). Fourier Transform (FT). Examples: FT of single pulse, trigonometric, exponential and Gaussian functions. FT of derivatives, Inverse FT, Convolution theorem. Properties of FT s (translation, change of scale, complex conjugation, etc.). Solution of one dimensional Wave Equation using FT. Fourier Sine Transform (FST) and Fourier Cosine Transform (FCT).

(12 Lectures)

Unit 3

Laplace Transforms: Laplace Transform (LT) of Elementary functions. Properties of LTs: Change of Scale Theorem, Shifting Theorem. LTs of 1st and 2nd order Derivatives and Integrals of Functions, Derivatives and Integrals of LTs. LT of Unit Step function, Periodic Functions. Convolution Theorem. Inverse LT. Application of Laplace Transforms to 2nd order Differential Equations, Coupled differential equations of 1st order. Solution of 1-D heat equation (semi-infinite bar) using LT.

(15 Lectures)

Unit 4

Dirac delta function: Definition and properties. Representation of Dirac delta function as a Fourier Integral. Laplace and Fourier Transform of Dirac delta function.

(3 Lectures)

Practical: 60 Hours

The aim of this Lab is to use the computational methods to solve physical problems. The course will consist of practical sessions and lectures on the related theoretical aspects of the Laboratory course. Evaluation done not only on the basis of programming but also on the basis of formulating the problem. **At least ten** programs must be attempted taking at least one from each programming section. The program list is only suggestive and students should be encouraged to do more problems.

C⁺⁺/C/Scilab/Python based simulations experiments on Mathematical Physics problems like

1. Boundary Value Problems :
 - A. Solution to Ordinary Differential equation (Boundary Value Problems using finite Difference and shooting methods) :
 - i. Solve $y''(x) + y(x) = 0$ with $y(0) = 1, y(\pi/2) = 1$ for $0 < x < \pi$.
 - ii. Solve for the steady state concentration profile $y(x)$ in the reaction-diffusion problem given by Solve $y''(x) - y(x) = 0$ with $y(0) = 1, y'(1) = 0$.

- B. Solution to Partial Differential equation: Finite Difference and Crank-Nicholson methods to solve Laplace equation, wave equation, and Heat Equation.

2. Gauss Quadrature Integration Method : Gauss Legendre, Gauss Laguerre and Gauss Hermite. :
 - i. Verification of Orthogonality of Legendre Polynomials.

$$\int_{-1}^{+1} P_n(\mu) P_m(\mu) d\mu = \frac{2}{(2n+1)} \delta_{n,m}$$

- ii. Complex analysis: Integrate $\int_0^{\infty} \frac{1}{(x^2+2)} dx$ numerically using Gauss Laguerre method and check with contour integration.

3. Dirac Delta Function: representations of Dirac delta function as a limiting sequence of functions. Verify the properties of Dirac Delta function. e.g. Evaluate

$\frac{1}{\sqrt{2\pi\sigma^2}} \int \exp\left(\frac{-(x-2)^2}{2\sigma^2}\right) (x+3) dx$, for $\sigma = 1, 0.1, 0.01$ and show that it tends to 5. Use Hermite Gauss quadrature method and also Simpson method with appropriate limits.

4. Fourier Series:
Evaluate the Fourier coefficients of a given periodic function (e.g. square wave, triangle wave, half wave and full wave rectifier etc.)
5. Weighted Least square fitting of given data (x,y) with known error/uncertainty-values using user defined function.
6. Integral transform:
 - i. Discrete and Fast Fourier Transform of given function in tabulated or mathematical form e.g function $\exp(-x^2)$.
 - ii. Perform circuit analysis of a general LCR circuit using Laplace's transform.

References for Theory:

Essential Readings:

1. Mathematical Methods for Physics and Engineers, K.F Riley, M.P. Hobson and S. J. Bence, 3rd ed., 2006, Cambridge University Press
2. Complex Variables and Applications, J.W.Brown& R.V.Churchill, 7th Ed. 2003, Tata McGraw-Hill.
3. Laplace Transform: Schaum's Outline, M.R> Spiegel, McGraw Hill Education.
4. Complex Variables: Schaum's Outline, McGraw Hill Education (2009).
5. Fourier Analysis and Its Applications (Wadsworth and Brooks/Cole Mathematics Series), Gerald B. Folland, Thomson Brooks/Cole (1992).

Additional Readings:

1. Mathematics for Physicists, P.Dennery and A.Krzywicki, 1967, Dover Publications.
2. Complex Variables, A.S.Fokas & M.J.Ablowitz, 8th Ed., 2011, Cambridge Univ. Press.
3. Mathematical Physics with Applications, Problems and Solutions, V. Balakrishnan, Ane Books (2017).
4. Fourier Analysis with Applications to Boundary Value Problems: : Schaum Outline Series, M. R Spiegel, McGraw Hill Education (1974).
5. Fourier Transform and its Applications, 2nd Ed., Ronald New Bold Bracewell, McGraw Hill (1978).

References for Laboratory Work:

1. An introduction to computational Physics, T.Pang, 2nd Edn.,2006, Cambridge Univ. Press
2. Applied numerical analysis, Cutis F. Gerald and P.O. Wheatley, Pearson Education, India (2007).
3. Friendly Introduction to Numerical Analysis, Brian Bradie, Pearson Education (2007).
4. Introduction to Numerical Analysis, S.S. Sastry, 5th Edn., PHI Learning Pvt. Ltd. (2012).
5. Partial Differential Equations for Scientists and Engineers, S.J. Farlow, Dover Publications (1993).