DSE-2 (ii): Discrete Mathematics

Total Marks: 100 (Theory: 75 + Internal Assessment: 25)
Workload: 5 Lectures, 1 Tutorial (per week) **Credits:** 6 (5+1)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The course aims at introducing the concepts of ordered sets, lattices, sublattices and homomorphisms between lattices. It also includes introduction to modular and distributive lattices along with complemented lattices and Boolean algebra. Then some important applications of Boolean algebra are discussed in switching circuits. The second part of this course deals with introduction to graph theory, paths and circuits, Eulerian circuits, Hamiltonian graphs and finally some applications of graphs to shortest path algorithms.

Course Learning outcomes: After the course, the student will be able to:

i) Understand the notion of ordered sets and maps between ordered sets.
ii) Learn about lattices, modular and distributive lattices, sublattices and homomorphisms between lattices.
iii) Become familiar with Boolean algebra, Boolean homomorphism, Karnaugh diagrams, switching circuits and their applications.
iv) Learn about basics of graph theory, including Eulerian graphs, Hamiltonian graphs.
v) Learn about the applications of graph theory in the study of shortest path algorithms.

Unit 1: Ordered Sets
Definitions, Examples and basic properties of ordered sets, Order isomorphism, Hasse diagrams, Dual of an ordered set, Duality principle, Maximal and minimal elements, Building new ordered sets, Maps between ordered sets.

Unit 2: Lattices
Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms; Definitions, Examples and properties of modular and distributive lattices, The $M_3 - N_5$ theorem with applications, Complemented lattice, Relatively complemented lattice, Sectionally complemented lattice.

Unit 3: Boolean Algebras and Switching Circuits
Boolean algebras, De Morgan's laws, Boolean homomorphism, Representation theorem; Boolean polynomials, Boolean polynomial functions, Disjunctive normal form and conjunctive normal form, Minimal forms of Boolean polynomial, Quine–McCluskey method, Karnaugh diagrams, Switching circuits and applications of switching circuits.

Unit 4: Graph Theory
Introduction to graphs, Königsberg bridge problem, Instant insanity game; Definition, examples and basic properties of graphs, Subgraphs, Pseudographs, Complete graphs, Bipartite graphs, Isomorphism of graphs, Paths and circuits, Eulerian circuits, Hamiltonian cycles, Adjacency matrix, Weighted graph, Travelling salesman problem, Shortest path, Dijkstra's algorithm.

References:

1. Davey, B. A., & Priestley, H. A. (2002). *Introduction to Lattices and Order* (2nd ed.). Cambridge University press, Cambridge.
2. Goodaire, Edgar G., & Parmenter, Michael M. (2011). *Discrete Mathematics with Graph Theory* (3rd ed.). Pearson Education (Singapore) Pvt. Ltd. Indian Reprint.
3. Lidl, Rudolf & Pilz, Gunter. (2004). *Applied Abstract Algebra* (2nd ed.), Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint.

Additional Reading:

i. Rosen, Kenneth H. (2012). *Discrete Mathematics and its Applications, with Combinatorics and Graph Theor*y. (7th ed.). McGraw-Hill Education. Indian Reprint.

Teaching Plan (DSE-2 (ii): Discrete Mathematics):

Weeks 1 and 2: Definitions, Examples and basic properties of ordered sets, Order isomorphism, Hasse diagrams, dual of an ordered set, Duality principle, Maximal and minimal elements, Building new ordered sets, Maps between ordered sets.

[1] Chapter 1 (Sections 1.1 to 1.5, Sections 1.14 to 1.26, and Sections 1.34 to 1.36).
[3] Chapter 1 [Section 1 (1.1 to 1.3)].

Weeks 3 and 4: Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms.

[1] Chapter 2 (Sections 2.1 to 2.19).
[3] Chapter 1 [Section 1 (1.5 to 1.20)].

Week 5: Definitions, Examples and properties of Modular and distributive lattices.

[1] Chapter 4 (Sections 4.1 to 4.9).
[3] Chapter 1 [Section 2 (2.1 to 2.6).

Week 6: $M_3 - N_5$ theorem with applications, Complemented lattice, Relatively complemented lattice, Sectionally complemented lattice.

[1] Chapter 4 (Sections 4.10 and 4.11).
[3] Chapter 1 [Section 2 (2.7 to 2.14)].

Weeks 7 and 8: Boolean algebras, De Morgan's laws, Boolean homomorphism, representation theorem, Boolean polynomials, Boolean polynomial functions, Disjunctive normal form and conjunctive normal form.

[3] Chapter 1 (Sections 3 and 4).

Week 9: Minimal forms of Boolean polynomial, Quine–McCluskey method, Karnaugh diagrams.

[3] Chapter 1 (Section 6).

Week 10: Switching circuits and applications of switching circuits.

[3] Chapter 2 (Sections 7 and 8).

Weeks 11 and 12: Introduction to graphs, Königsberg bridge problem, Instant insanity game. Definition, Examples and basic properties of graphs, Subgraphs, Pseudographs, Complete graphs, Bipartite graphs, Isomorphism of graphs.

[2] Chapter 9 [Sections 9.1, 9.2 (9.2.1, 9.2.7) and 9.3].

Weeks 13 and 14: Paths and circuits, Eulerian circuits, Hamiltonian cycles, Adjacency matrix, Weighted graph, Travelling salesman problem, shortest path, Dijkstra's algorithm.

[2] Chapter 10 [Sections 10.1 to 10.4 (10.4.1 to 10.4.3)].

Facilitating the Achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1.	Understand the notion of ordered sets and maps between ordered sets.	(i) Each topic to be explained with examples. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations.	• Presentations and participation in discussions. • Assignments and class tests. • Mid-term examinations. • End-term examinations.
2.	Learn about lattices, modular and distributive lattices, sublattices and homomorphisms between lattices.		
3.	Become familiar with Boolean algebra, Boolean homomorphism, Karnaugh diagrams, switching circuits and their applications.		
4.	Learn about basics of graph theory, including Eulerian graphs, Hamiltonian graphs. Learn about the applications of graph theory in the study of shortest path algorithms.		

Keywords: Boolean algebra, Lattices, Graphs, Modularity, Ordered sets, Paths and circuits, Shortest path algorithms, Switching circuits.