BMATH204: Differential Equations

Total Marks: 150 (Theory: 75, Internal Assessment: 25 and Practical: 50)
Workload: 4 Lectures, 4 Practicals (per week) **Credits:** 6 (4+2)
Duration: 14 Weeks (56 Hrs. Theory + 56 Hrs. Practical) **Examination:** 3 Hrs.

Course Objectives: The main objective of this course is to introduce the students to the exciting world of differential equations, mathematical modeling and their applications.

Course Learning Outcomes: The course will enable the students to:

i) Learn basics of differential equations and mathematical modeling.
ii) Formulate differential equations for various mathematical models.
iii) Solve first order non-linear differential equations and linear differential equations of higher order using various techniques.
iv) Apply these techniques to solve and analyze various mathematical models.

Unit 1: Differential Equations and Mathematical Modeling
Differential equations and mathematical models, Order and degree of a differential equation, Exact differential equations and integrating factors of first order differential equations, Reducible second order differential equations, Applications of first order differential equations to acceleration-velocity model, Growth and decay model.

Unit 2: Population Growth Models
Introduction to compartmental models, Lake pollution model (with case study of Lake Burley Griffin), Drug assimilation into the blood (case of a single cold pill, case of a course of cold pills, case study of alcohol in the bloodstream), Exponential growth of population, Limited growth of population, Limited growth with harvesting.

Unit 3: Second and Higher Order Differential Equations
General solution of homogeneous equation of second order, Principle of superposition for a homogeneous equation; Wronskian, its properties and applications, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation, Method of undetermined coefficients, Method of variation of parameters, Applications of second order differential equations to mechanical vibrations.

Unit 4: Analysis of Mathematical Models
Interacting population models, Epidemic model of influenza and its analysis, Predator-prey model and its analysis, Equilibrium points, Interpretation of the phase plane, Battle model and its analysis.

References:

1. Barnes, Belinda & Fulford, Glenn R. (2015). *Mathematical Modelling with Case Studies, Using* Maple *and* MATLAB (3rd ed.). CRC Press, Taylor & Francis Group.
2. Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). *Differential Equation and Boundary Value Problems*: *Computing and Modeling* (5th ed.). Pearson Education.
3. Ross, Shepley L. (2004). *Differential Equations* (3rd ed.). John Wiley & Sons. India

Additional Reading:

i. Ross, Clay C. (2004). *Differential Equations*: *An Introduction with Mathematica*® (2nd ed.). Springer.

Practical / Lab work to be performed in a Computer Lab:

Modeling of the following problems using Mathematica /MATLAB/Maple/Maxima/Scilab etc.

1. Plotting of second and third order respective solution family of differential equation.
2. Growth and decay model (exponential case only).
3. (i) Lake pollution model (with constant/seasonal flow and pollution concentration).
   (ii) Case of single cold pill and a course of cold pills.
   (iii) Limited growth of population (with and without harvesting).
4. (i) Predatory-prey model (basic Volterra model, with density dependence, effect of DDT, two prey one predator).
   (ii) Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
   (iii) Battle model (basic battle model, jungle warfare, long range weapons).
5. Plotting of recursive sequences, and study of the convergence.
6. Find a value $m \in \mathbb{N}$ that will make the following inequality holds for all $n > m$:
   $(i)\ \left|\sqrt[n]{0.5} - 1\right| < 10^{-3}$, $\quad (ii)\ \left|\sqrt[n]{n} - 1\right| < 10^{-3}$,
   $(iii)\ (0.9)^n < 10^{-3}$, $\quad (iv)\ \frac{2^n}{n!} < 10^{-7}$, etc.
7. Verify the Bolzano–Weierstrass theorem through plotting of sequences and hence identify convergent subsequences from the plot.
8. Study the convergence/divergence of infinite series of real numbers by plotting their sequences of partial sum.
9. Cauchy's root test by plotting *n*th roots.
10. D'Alembert's ratio test by plotting the ratio of *n*th and (*n*+1)th term of the given series of positive terms.
11. For the following sequences $\langle a_n \rangle$, given $\varepsilon = \frac{1}{2^k}, p = 10^j, k = 0,1,2,\ldots; j = 1,2,3,\ldots$
    Find $m \in \mathbb{N}$ such that
    $(i)\ \left|a_{m+p} - a_m\right| < \varepsilon, \quad (ii)\ \left|a_{2m+p} - a_{2m}\right| < \varepsilon,$
    where a_n is given as:
    $$(a)\ \frac{n+1}{n}, \quad (b)\ \frac{1}{n}, \quad (c)\ 1 - \frac{1}{2} + \frac{1}{3} - \cdots + \frac{(-1)^{n-1}}{n}$$
    $$(d)\ \frac{(-1)^n}{n}, \quad (e)\ 2^{-n}n^2, \quad (f)\ 1 + \frac{1}{2!} + \cdots + \frac{1}{n!}.$$
12. For the following series $\sum a_n$, calculate
    $(i)\ \left|\frac{a_{n+1}}{a_n}\right|, \quad (ii)\ |a_n|^{\frac{1}{n}}$, for $n = 10^j, j = 1,2,3,\ldots$,
    and identify the convergent series, where a_n is given as:
    $$(a)\left(\frac{1}{n}\right)^{1/n}, \quad (b)\frac{1}{n}, \quad (c)\ \frac{1}{n^2}, \quad (d)\left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}},$$
    $$(e)\ \frac{n!}{n^n}, \quad (f)\ \frac{n^3 + 5}{3^n + 2}, \quad (g)\ \frac{1}{n^2 + n}, \quad (\square)\frac{1}{\sqrt{n+1}},$$
    $$(i)\cos n, \quad (j)\frac{1}{n\log n}, \quad (k)\frac{1}{n(\log n)^2}.$$

Teaching Plan (Theory of BMATH204: Differential Equations):

Weeks 1 and 2: Differential equations and mathematical models, Order and degree of a differential equation, Exact differential equations and integrating factors of first order differential equations, Reducible second order differential equations.

[2] Chapter 1 (Sections 1.1 and 1.6).
[3] Chapter 2.

Week 3: Application of first order differential equations to acceleration-velocity model, Growth and decay model.

[2] Chapter 1 (Section 1.4, Pages 35 to 38), and Chapter 2 (Section 2.3).
[3] Chapter 3 (Section 3.3, A and B with Examples 3.8, 3.9).

Week 4: Introduction to compartmental models, Lake pollution model (with case study of Lake Burley Griffin).

[1] Chapter 2 (Sections 2.1, 2.5 and 2.6).

Week 5: Drug assimilation into the blood (case of a single cold pill, case of a course of cold pills, Case study of alcohol in the bloodstream).

[1] Chapter 2 (Sections 2.7 and 2.8).

Week 6: Exponential growth of population, Density dependent growth, Limited growth with harvesting.

[1] Chapter 3 (Sections 3.1 to 3.3).

Weeks 7 to 9: General solution of homogeneous equation of second order, Principle of superposition for a homogeneous equation; Wronskian, its properties and applications; Linear homogeneous and non-homogeneous equations of higher order with constant coefficients; Euler's equation.

[2] Chapter 3 (Sections 3.1 to 3.3).

Weeks 10 and 11: Method of undetermined coefficients, Method of variation of parameters; Applications of second order differential equations to mechanical vibrations.

[2] Chapter 3 (Sections 3.4 (Pages 172 to 177) and 3.5).

Weeks 12 to 14: Interacting population models, Epidemic model of influenza and its analysis, Predator-prey model and its analysis, Equilibrium points, Interpretation of the phase plane, Battle model and its analysis.

[1] Chapter 5 (Sections 5.1, 5.2, 5.4 and 5.9), and Chapter 6 (Sections 6.1 to 6.4).

Facilitating the Achievement of Course Learning Outcomes

Unit No.	Course Learning Outcomes	Teaching and Learning Activity	Assessment Tasks
1.	Learn basics of differential equations and mathematical modeling.	(i) Each topic to be explained with examples and illustrated on computers using Mathematica /MATLAB /Maple/Maxima/Scilab. (ii) Students to be involved in discussions and encouraged to ask questions. (iii) Students to be given homework/assignments. (iv) Students to be encouraged to give short presentations.	• Presentations and participation in discussions. • Assignments and class tests. • Mid-term examinations. • Practical and viva-voce examinations. • End-term examinations.
2.	Formulate differential equations for various mathematical models.		
3.	Solve first order non-linear differential equations and linear differential equations of higher order using various techniques.		
4.	Apply these techniques to solve and analyze various mathematical models.		

Keywords: Battle model, Epidemic model, Euler's equation, Exact differential equation, Integrating factor, Lake pollution model, Mechanical vibrations, Phase plane, Predator-prey model, Wronskian and its properties.